NUMBER OF SPOTLESS DAYS IN RELATION TO THE TIMING AND SIZE OF SUNSPOT CYCLE MINIMUM

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ABSTRACT

Sunspot cycle (SC) 24, the present ongoing SC, is now in its 9th year, having had a minimum in annual sunspot number (SSN) in 2008, measuring 4.2, and a minimum in 'smoothed' SSN in December 2008, measuring 2.2. Its maximum annual SSN occurred in 2014, measuring 113.3, and its maximum smoothed SSN occurred in April 2014, measuring 116.4. Following smoothed SSN maximum (SSN max), the first spotless day (FSD) during the decline of SC24 was reported in July 2014, a mere 3 months following SC24's maximum SSN amplitude, an interval shorter than the 13 months found for SC12 and the 8 months for SC14. Through December 2016, some 27 spotless days (1 in 2014 and 26 in 2016) have been seen, with more to come as SC25's epoch of minimum approaches. Examined in this study are the following three factors: (1) the variation in number of spotless days (NSD) relative to the epochs of sunspot minimum (SSN min) and SSN max, (2) the association between the maximum NSD and the SSN min, and (3) cyclic variations of selected spotless day-associated parameters. It is suggested that SC25's epoch of SSN min likely will occur sometime in 2020 or later and that SC24's NSD behavior represents a return to that experienced during SC12 and SC14. Hence, another prolonged minimum, like that experienced for SC23/24, might well occur for SC24/25. For comparison, SC24's interval of spotless days from first to last spotless day bounding SSN min extended 84 months and totaled 816 spotless days, with 265 spotless days reported during the year of annual SSN min.

INTRODUCTION

Sunspots are the dark, cool regions of enhanced magnetic field strength found on the Sun that vary in number, size, and magnetic complexity over the course of the sunspot cycle (SC) (Wilson 2015). Indeed, it was on the basis of the annual number of spotless days (NSD) and number of newly appearing 'clusters of spots' that Samuel Heinrich Schwabe (1844) first showed that the Sun's activity varies from a sunspot minimum (SSN min) to a sunspot maximum (SSN max) and then to another minimum over a period of about a decade. Near SSN min, the NSD increases to a NSD maximum (NSD max), while near SSN max they rarely occur, being essentially nonexistent. On the other hand, near SSN min, the number of newly appearing clusters of spots (akin to the number of sunspot 'groups') is minimal, while near SSN max, they are maximal. Furthermore, Wilson (1995) showed that the occurrence of the first spotless day (FSD) during the declining portion of the SC can be used as a predictor for the timing and size of the following SSN min (cf. Harvey and White 1999, Wilson and Hathaway 2005, 2006).

SC24, the present ongoing SC, had its SSN min in 2008, measuring 4.2 in terms of the annual average (i.e., January–December). Its SSN max occurred in 2014, measuring 113.3 in terms of the annual average. Since 2014, the annual average of SSN has decreased, measuring 69.8 in 2015 and 39.9 in 2016. In terms of the 12-month moving average of SSN (also called the 13-month running mean or simply 'smoothed' SSN), SC24 had its minimum in December 2008 (this being the 'epoch' of SSN min, (Em)), measuring 2.2, and its maximum in April 2014 (this being the epoch of SSN max, (EM)), measuring 116.4. In June 2016 (the last available smoothed SSN at the time of writing this manuscript), the 12-month moving average of SSN had decreased to 41.5.

In terms of NSD, the FSD for SC24 occurred in November 2004 during the decline of SC23 (i.e., post EM cycle 23), and the last spotless day (LSD) occurred in January 2011 post Em for SC24 and prior to its EM, thus, spanning some 84 months and totaling 816 in number, these values being considerably longer and larger than those observed since SC15. In comparison to other modern era SCs (i.e., SC10–present), SC24's NSD is only the third largest, being smaller than SC12's 1,027 spotless days and SC15's 1,018 spotless days. In terms of the length from FSD to LSD, SC24 ranks as the fifth longest, shorter than SC12's 124 months, SC15's 120 months, SC14's 116 months, and SC10's 107 months. (Recall, FSD for cycle n +1 is defined as the first spotless day post EM for cycle n, and LSD for cycle n +1 is defined as the last spotless day prior to EM for cycle n +1.)

Following SC24's EM, the FSD for SC25, the next SC, made its appearance in July 2014, a mere 3 months following SC24's EM. Through December 2016, some 27 spotless days have been seen, with more to come as SC24 progresses towards SC25's Em, not expected to occur until about 2020 or later (Wilson 2015).

SC24 had a maximum amplitude that is the smallest since SC14 and had both an extended interval from FSD to LSD, as well as a large NSD. This has fostered the idea that the Sun might be entering another lull of activity, similar to the one called the 'Dalton minimum' that occurred with SC05 and SC06 (cf. Russell, Luhmann, and Jian 2010; Mordvinov and Kramynin 2010; Nielsen and Kjeldsen 2011; Zięba and Nieckarz 2012; Clette and Lefèvre 2012; Hady 2013; Zięba and Nieckarz 2014; McCracken and Beer 2014; and Mőrner 2015). In this article, the NSD is examined relative to the timing and size of cycle minimum and maximum, with a look toward predicting the occurrence of Em for SC25.

METHODS AND MATERIALS

SSN and NSD values were obtained using the newly revised SSN data taken from the "Sunspot Index and Long-term Solar Observations" webpage available online through the Solar Influences Data Analysis Center website <www.sidc.oma.be/index.php3>. Linear regression analysis, runs testing, and nonparametric analyses (i.e., Kendall's τ and Fisher's exact test for 2×2 contingency tables) are employed to evaluate the inferred relationships.

RESULTS AND DISCUSSION

Figure 1 displays the annual values of (a) SSN and (b) NSD for the interval 1849–2016, spanning the declining portions of SC09–SC24. Clearly, the occurrence of NSD max has always peaked with the occurrence of SSN min. The interval encompassing SC12–SC14 appears to represent a brief lull in solar activity (based on SSN), especially in comparison to the previous SC09–SC11 and subsequent SCs. In terms of SSN max, SC24 appears strikingly similar to that of SC12 and SC14.

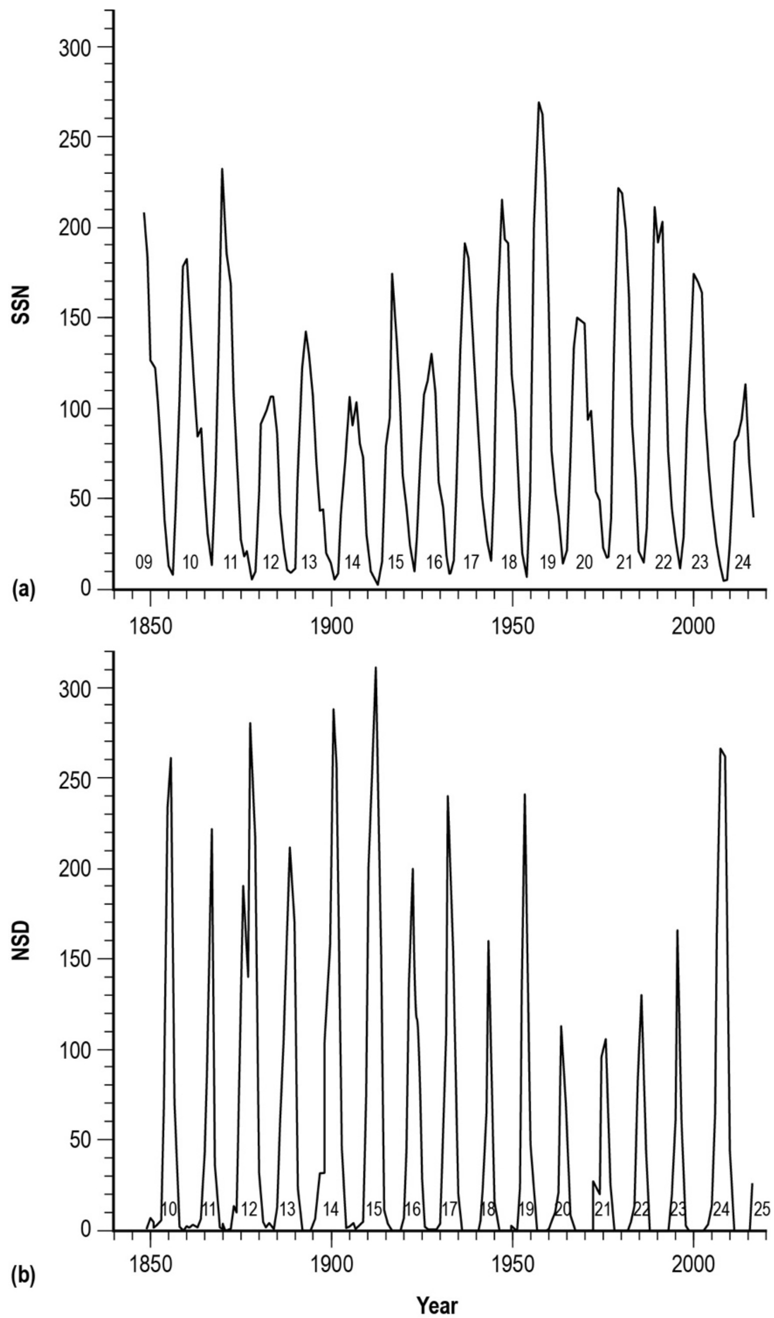

Figure 1. Annual variation of (a) sunspot number (SSN) and (b) number of spotless days (NSD) for the modern era of sunspot observations. The sunspot cycle (SC) number is identified for each cycle.

Regarding NSD, its rise to NSD maximum (NSD max) is shown to always precede the occurrence of SSN min, peaking at SSN min and then falling to zero prior to the occurrence of SSN max for all modern era SCs. Clearly, the size of SC24's NSD max, although quite large in comparison to recent SCs (SC20–SC23), is only the fourth largest NSD max (265 spotless days at SSN min), being below those of SC15 (311), SC14 (287), and SC12 (280). For convenience, Table 1 is included to provide the listing of the specific annual values of SSN and NSD plotted in Figure 1.

Table 1. Annual mean sunspot number (SSN) and annual number of spotless days (NSD), 1848–2016 (Continued).

Year	SSN	NSD	Comment
1848	208.3	#	SC09 SSN max
1849	182.5	1	
1850	126.3	7	
1851	122.0	0	
1852	102.7	4	
1853	74.1	6	
1854	39.0	70	
1855	12.7	234	
1856	8.2	261	SC10 SSN min, NSD max
1857	43.4	70	
1858	104.4	2	
1859	178.3	0	
1860	182.2	0	SC10 SSN max
1861	146.6	2	
1862	112.1	3	
1863	83.5	2	
1864	89.2	7	
1865	57.8	44	
1866	30.7	86	
1867	13.9	222	SC11 SSN min, NSD max
1868	62.8	37	
1869	123.6	2	
1870	232.0	0	SC11 SSN max
1871	185.3	0	
1872	169.2	0	
1873	110.1	14	
1874	74.5	12	
1875	28.3	131	
1876	18.9	190	
1877	20.7	140	
1878	5.7	280	SC12 SSN min, NSD max
1879	10.0	217	
1880	53.7	32	
1881	90.5	5	
1882	99.0	2	
1883	106.1	4	SC12 SSN max
1884	105.8	0	
1885	86.3	13	
1886	42.4	62	
1887	21.8	104	
1888	11.2	150	

Table 1. Annual mean sunspot number (SSN) and annual number of spotless days (NSD), 1848–2016 (Continued).

Year	SSN	NSD	Comment
1889	10.4	212	SC13 SSN min, NSD max
1890	11.8	171	
1891	59.5	24	
1892	121.7	0	
1893	142	0	SC13 SSN max
1894	130	0	
1895	106.6	1	
1896	69.4	7	
1897	43.8	32	
1898	44.4	32	
1899	20.2	104	
1900	15.7	158	
1901	4.6	287	SC14 SSN min, NSD max
1902	8.5	257	
1903	40.8	45	
1904	70.1	1	
1905	105.5	2	SC14 SSN max
1906	90.1	4	
1907	102.8	0	
1908	80.9	3	
1909	73.2	5	
1910	30.9	75	
1911	9.5	201	
1912	6.0	254	
1913	2.4	311	SC15 SSN min, NSD max
1914	16.1	153	
1915	79.0	12	
1916	95.0	4	
1917	173.6	0	SC15 SSN max
1918	134.6	0	
1919	105.7	0	
1920	62.7	7	
1921	43.5	46	
1922	23.7	134	
1923	9.7	200	SC16 SSN min, NSD max
1924	27.9	116	
1925	74.0	29	
1926	106.5	2	
1927	114.7	0	
1928	129.7	0	SC16 SSN max
1929	108.2	0	
1930	59.4	3	
1931	35.1	43	
1932	18.6	108	
1933	9.2	240	SC17 SSN min, NSD max
1934	14.6	154	
1935	60.2	20	
1936	132.8	0	
1937	190.6	0	SC17 SSN max
1938	182.6	0	
1939	148.0	0	

Table 1. Annual mean sunspot number (SSN) and annual number of spotless days (NSD), 1848–2016 (Continued).

Year	SSN	NSD	Comment
1940	113.0	0	
1941	79.2	5	
1942	50.8	24	
1943	27.1	65	
1944	16.1	159	SC18 SSN min, NSD max
1945	55.3	16	
1946	154.3	0	
1947	214.7	0	SC18 SSN max
1948	193.0	0	
1949	190.7	0	
1950	118.9	3	
1951	98.3	0	
1952	45.0	23	
1953	20.1	131	
1954	6.6	241	SC19 SSN min, NSD max
1955	54.2	48	
1956	200.7	0	
1957	269.3	0	SC19 SSN max
1958	261.7	0	
1959	225.1	0	
1960	159.0	0	
1961	76.4	6	
1962	53.4	10	
1963	39.9	21	
1964	15.0	112	SC20 SSN min, NSD max
1965	22.0	70	
1966	66.8	8	
1967	132.9	0	
1968	150.0	0	SC20 SSN max
1969	149.4	0	
1970	148.0	0	
1971	94.4	0	
1972	97.6	0	
1973	54.1	27	
1974	49.2	20	
1975	22.5	96	
1976	18.4	105	SC21 SSN min, NSD max
1977	39.3	25	
1978	131.0	0	
1979	220.1	0	SC21 SSN max
1980	218.9	0	
1981	198.9	0	
1982	162.4	0	
1983	91.0	4	
1984	60.5	13	
1985	20.6	83	
1986	14.8	129	SC22 SSN min, NSD max
1987	33.9	44	
1988	123.0	0	
1989	211.1	0	SC22 SSN max
1990	191.8	0	

Table 1. Annual mean sunspot number (SSN) and annual number of spotless days (NSD), 1848–2016 (Continued).

Year	SSN	NSD	Comment
1991	203.3	0	
1992	133.0	0	
1993	76.1	0	
1994	44.9	19	
1995	25.1	61	
1996	11.6	165	SC23 SSN min, NSD max
1997	28.9	61	
1998	88.3	3	
1999	136.3	0	
2000	173.9	0	SC23 SSN max
2001	170.4	0	
2002	163.6	0	
2003	99.3	0	
2004	65.3	3	
2005	45.8	13	
2006	24.7	65	
2007	12.6	163	
2008	4.2	265	SC24 SSN min, NSD max
2009	4.8	262	
2010	24.9	44	
2011	80.8	1	
2012	84.5	0	
2013	94.0	0	
2014	113.3	1	SC24 SSN max
2015	69.8	0	
2016	39.9	26	

means missing days during year; NSD uncertain

A comparison of SSN min against NSD max is shown in Figure 2. Based on linear regression analysis, the relationship between the two parameters is inferred to be $y = 24.536 - 0.068x$, where y is SSN min, and x is NSD max. The inferred regression has a coefficient of correlation $r = -0.925$ and a coefficient of determination $r^2 = 0.856$, meaning that the inferred regression can explain about 85.6% of the variance in SSN min, based on NSD max being the independent variable. The inferred standard error of estimate is $se = 1.844$ units of SSN, and the inferred relationship has a confidence level cl >99.9%. Instead, based on Kendall's τ (Gibbons 1993), a nonparametric technique, the association between SSN min and NSD max is computed to be about $\tau = -0.829$, having a Z-statistic equal to about 4.308, suggesting a very strong association to exist between the two parameters. Also given in Figure 2 is the result of Fisher's exact test for a 2×2 contingency table (Everitt 1977), determined by the medians of SSN min and NSD max (respectively, the horizontal and vertical lines shown in Figure 2). Hence, the probability P of obtaining the observed distribution, or one more suggestive of a departure from independence (i.e., chance), is computed to be $P = 0.00124$, a highly statistically significant result. Therefore, by assuming an approximate size of NSD max for a SC in advance, one can estimate the approximate size of its SSN min based on the inferred linear regression. As an example, presuming that SC25 will have NSD max = 265 (like that of SC24), one infers that there is a 90% probability that SSN min for SC25 will be about 6.5 ± 3.3. A larger NSD max suggests a lower SSN min, while a smaller NSD max suggests a larger SSN min.

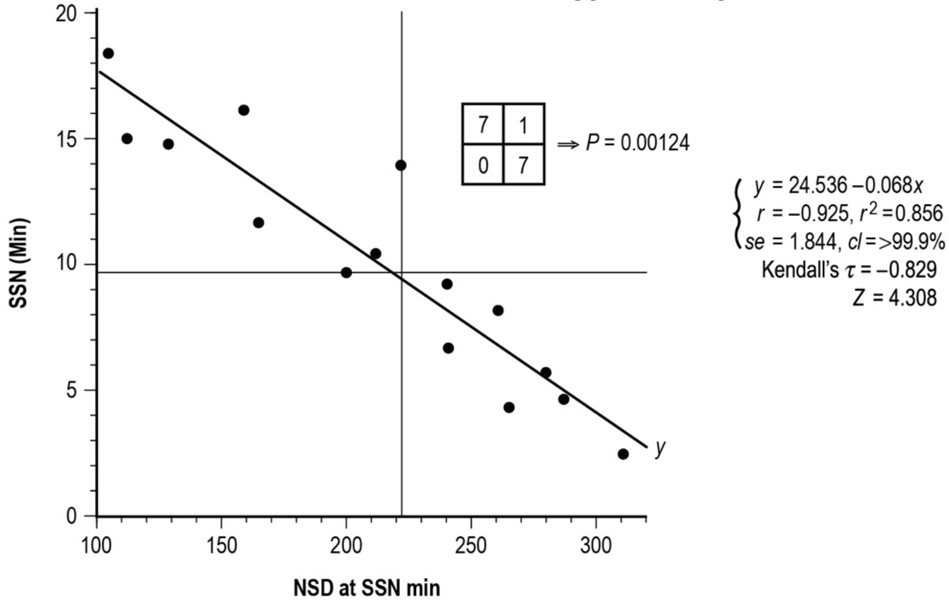

Figure 2. Scatterplot of sunspot number minimum (SSN min) versus the number of spotless days (NSD) at SSN min. The diagonal line is the inferred regression determined from linear regression analysis. The thin vertical and horizontal lines are the medians. The results of linear analysis and nonparametric analyses (both Kendall's τ and Fisher's exact test for 2 × 2 contingency tables) are given.

Figure 3(a) depicts NSD versus the elapsed time in years from SSN min (t) for $t = -5$ years to $t = +3$ years, in terms of the mean for SC10–SC24 (the thick line) and the greatest observed (GO) and least observed (LO) yearly values. On average, the mean NSD at $t = 0$ is 212.6 with GO = 311 (SC15) and LO = 105 (SC21). It should be noted that 10 of the 15 SCs have NSD ≥200 at $t = 0$, including SC10–SC17 and SC24; SC18–SC23 have NSD <200 at $t = 0$. For convenience, Table 2 is included to provide the yearly values, means, and standard deviation (*sd*) values for SC10–SC24 from $t = -5$ to $t = +3$. Figure 3(b) compares NSD for SC12 (dashed line), SC14 (thin line), and SC24 (thick line) for $t = -5$ to $t = +3$. All three SCs closely mimic each other from $t = -1$ to $t = +3$. Prior to $t = -1$, however, NSD for SC24 appears more closely to resemble that of SC14 rather than SC12. If SC14 can be used as a proxy for SC24, then this suggests that its cycle length will be about 12 years, inferring SSN min for SC25 in 2020. Figure 3(c) plots NSD versus elapsed time in years from SSN max (T) for $T = 0$ to $T = +8$ years, in terms of the mean for SC09–SC23 (the thick line) and the GO and LO yearly values. Also plotted are the three years for SC24 (filled circles) for $T = 0$ to $T = +2$ (i.e., 2014, 2015, and 2016; recall that NSD values post EM for cycle n are the NSD values for cycle $n + 1$, not cycle n). Noticeable is that at $T = +2$, the NSD for SC24 exceeds that of the GO value. The GO value for $T = +3$ years is 62 (SC12); so, if the year 2017 proves to be a year of rapid growth in NSD, then NSD might well exceed 62. For convenience, Table 3 is included to provide the yearly values, means, and *sd* values for SC09–SC23 at $T = 0$ to $T = +8$ and for SC24 at $T = 0$ to $T = +2$. (It should be noted that January 2017 had 10 spotless days.) Figure 3(d) compares NSD for SC12 (dashed line) and SC14 (thick line) for $T = 0$ to $T = +8$ and for SC24 (filled circles) for $T = 0$ to T

= +2. For T = 0 to T = +2, SC24 appears to more closely resemble that of SC12, suggesting that SSN min for SC25 might follow SC24's SSN max by about 6 years (i.e., 2020). (It should be noted that using SC14's NSD relative to SSN max as the template for SC24 suggests that SSN min for SC25 would be delayed until about 2022, inferring a cycle length of 14 years for SC24, a length never before seen in the modern era sunspot record.)

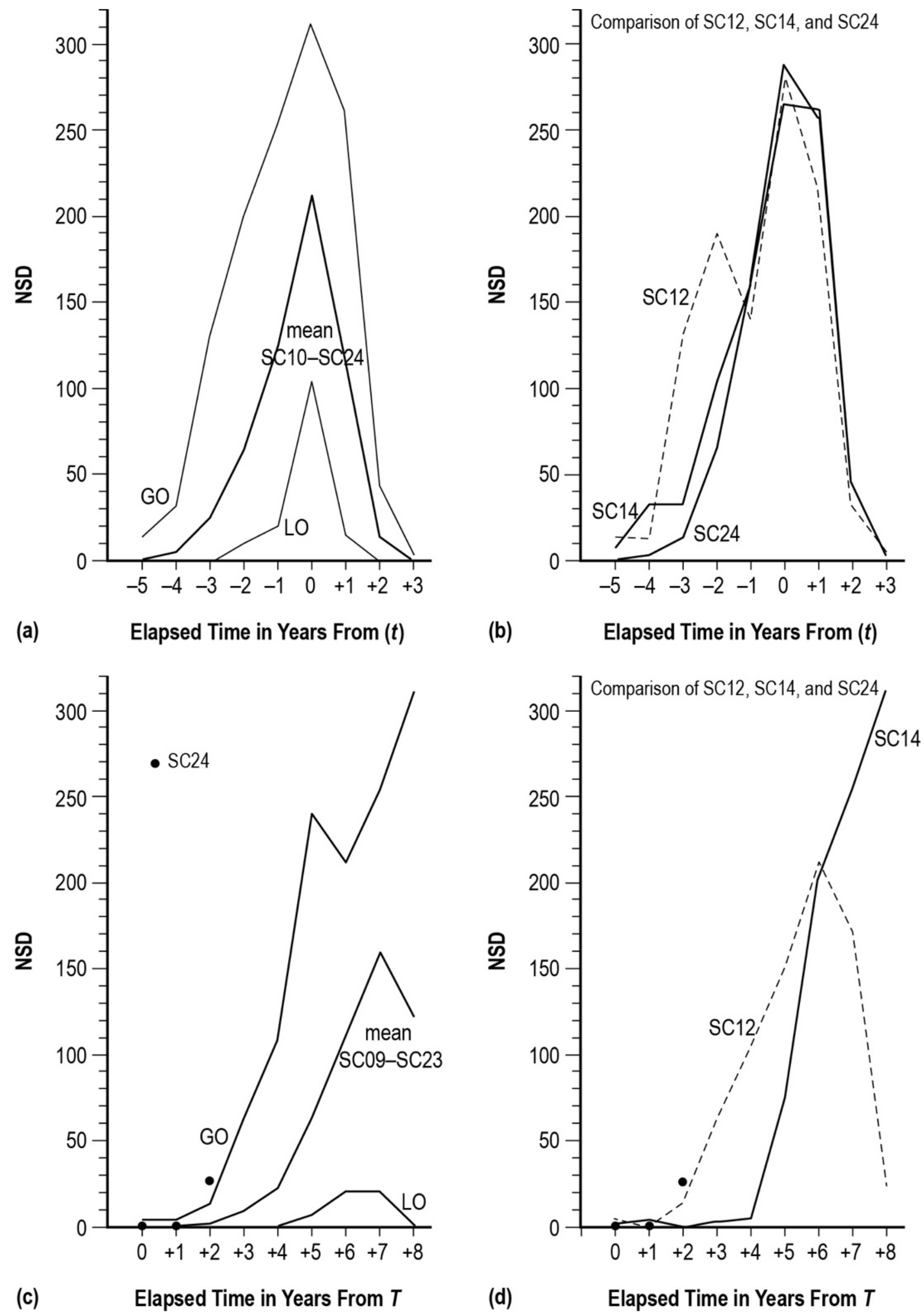

Figure 3. (a) The variation of the number of spotless days (NSD) in terms of the mean of sunspot cycles SC10–SC24, the lowest observed (LO) value and the greatest observed (GO) value for elapsed time (*t*) –5 years to +3 years relative to the elapsed time in years from

sunspot number minimum (SSN min) (*t*); (b) comparison of SC12, SC14 and SC24 number of spotless days (NSD) for *t* = –5 years to +3 years; (c) the variation of the NSD (Continued) in terms of the mean of SC10–SC24, the LO value and the GO value for elapsed time 0 to +8 years relative to the elapsed time in years from sunspot number maximum (SSN max) (*T*); and (d) comparison of SC12 and SC14 NSD for *T* = 0 to +8 years and SC24 NSD for *T* = 0 to +2 years.

Table 2. Number of spotless days (NSD) in vicinity of sunspot number minimum (SSN min) for elapsed time in years *t* = –5 to +3 years relative to SSN min occurrence (*t* = 0).

	NSD (*t*)								
Cycle	**–5**	**–4**	**–3**	**–2**	**–1**	**0**	**+1**	**+2**	**+3**
10	0	4	6	70	234	261	70	2	0
11	3	2	7	44	86	222	37	2	0
12	14	12	131	190	140	280	217	32	5
13	0	13	62	104	150	212	171	24	0
14	7	32	32	104	158	287	257	45	1
15	3	5	75	201	254	311	153	12	4
16	0	0	7	46	134	200	116	29	2
17	0	0	3	43	108	240	154	20	0
18	0	0	5	24	65	159	16	0	0
19	0	3	0	23	131	241	48	0	0
20	0	0	6	10	21	112	70	8	0
21	0	0	27	20	96	105	25	0	0
22	0	0	4	13	83	129	44	0	0
23	0	0	0	19	61	165	61	3	0
24	0	3	13	65	163	265	262	44	1
mean	1.8	4.9	25.2	65.1	125.6	212.6	113.4	14.7	0.9
sd	3.9	8.6	37.1	60.8	62.6	65.7	84.0	16.4	1.6

Table 3. Number of spotless days (NSD) from sunspot maximum (SSN max) for elapsed time in years *T* = 0 to +8 relative to SSN max occurrence (*T* = 0).

	NSD (*T*) for cycle *n* + 1								
Cycle	**0**	**+1**	**-2**	**+3**	**+4**	**+5**	**+6**	**+7**	**+8**
09	–	1	[illegible]	0	4	6	70	234	261
10	0	2	[illegible]	2	7	44	86	222	37
11	0	0	[illegible]	14	12	131	190	140	280
12	4	0	[illegible].3	62	104	150	212	171	24
13	0	0	[illegible]	7	32	32	104	158	287
14	2	4	[illegible]	3	5	75	201	254	311
15	0	0	[illegible]	7	46	134	200	116	4
16	0	0	[illegible]	43	108	240	154	20	0
17	0	0	[illegible]	0	5	24	65	159	16
18	0	0	[illegible]	3	0	23	131	241	58
19	0	0	[illegible]	0	6	10	21	112	70
20	0	0	[illegible]	0	0	27	20	96	105
21	0	0	[illegible]	0	4	13	83	129	44
22	0	0	[illegible]	0	0	19	61	165	61
23	0	0	[illegible]	0	3	13	65	163	265
mean	0.4	0.5	[illegible].8	9.4	22.4	62.7	110.9	158.7	121.5
sd	1.2	1.1	[illegible].7	18.3	36.3	69.4	66.0	62.2	120.0
24	1	0	[illegible]6	–	–	–	–	–	–

Figure 4 shows the variation of SSN using 12-month moving averages for the interval January 1989 through June 2016, spanning the declining portion of SC22 through the declining portion of SC24. The Em and EM are identified for each SC. Interesting is that all three SCs are double peaked, with maximum amplitude occurring with the first peak in SC22 but with the second peak in SC23 and SC24. Previously, Wilson (2015) showed that for SC24, the double peaking was related to hemispheric timing differences, with the first peak in SC24 being associated with the peak number of the northern hemispheric spots on the Sun and the second main peak being associated with the peak number of the southern hemispheric spots on the Sun. Across the top are the occurrences and values of NSD for SC23, SC24, and SC25, marking specific timing events and lengths. These events are FSD and LSD for each SC. From these events, one determines t_1, t_2, t_3, t_4, t_5, and t_6, where t_1 is the elapsed time in months from FSD (cycle n) to Em (cycle n), t_2 is the elapsed time in months from Em (cycle n) to LSD (cycle n), t_3 is the elapsed time in months from FSD (cycle n) to LSD (cycle n) or simply $t_1 + t_2$, t_4 is the elapsed time in months from LSD (cycle n) to FSD (cycle $n + 1$), t_5 is the elapsed time in months from FSD (cycle n) to FSD (cycle $n + 1$), and t_6 is the elapsed time from EM (cycle n) to FSD (cycle $n + 1$). For convenience, Table 4 is included to provide the Em, EM, FSD, and LSD dates for SC09–SC25 (when known), as well as the timing intervals $t_1 - t_6$ and the NSD for timing intervals $t_1 - t_3$.

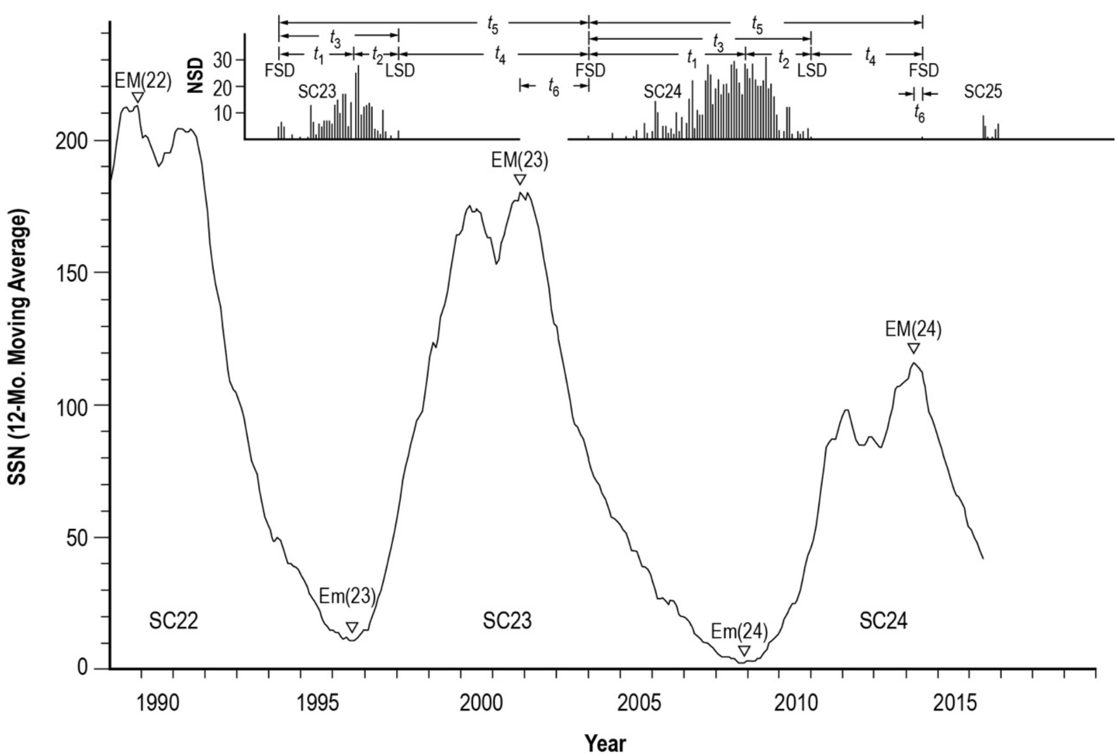

Figure 4. Bottom: The variation of 12-month moving average of monthly mean SSN for SC22–SC25. Epochs of minimum (Em) and epochs of maximum (EM) are identified. Top: The variation of monthly number of spotless days (NSD) for SC23–SC24. Also shown are t_1, t_2, t_3, t_4, t_5, and t_6, where t_1 is the elapsed time in months (t) from first spotless day (FSD) cycle n to epochs of minimum (Em) cycle n, t_2 is (t) from Em cycle n to last spotless day

(LSD) cycle *n*, t_3 is (*t*) from FSD (cycle *n*) to LSD (cycle *n*), t_4 (*t*) from LSD (cycle *n*) (Continued)
to FSD (cycle *n* + 1), t_5 is (*t*) from FSD (cycle *n*) to FSD (cycle *n* + 1), and t_6 is (*t*) from EM (cycle *n*) to FSD (cycle *n* + 1).

Table 4. First spotless day (FSD) and last spotless day (LSD) occurrences relative to the epochs of sunspot minimum (SSN min) (Em) and sunspot maximum (SSN max) (EM), number of months per timing interval, and NSD per timing interval.

Cycle	Em	EM	FSD	LSD	t_1	t_2	t_3	t_4	t_5	t_6	NSD (t_1)	NSD (t_2)	NSD (t_3)
09	1843-07	1848-02	–	–	–	–	–	–	–	15	–	–	–
10	1855-12	1860-02	1849-05?	1858-04	79	28	107	42	149	20	322	333	655
11	1867-03	1870-08	1861-10	1869-07	65	28	93	46	139	33	213	192	405
12	1878-12	1883-12	1873-05	1883-09	67	57	124	16	140	13	767	260	1027
13	1890-03	1894-01	1885-01	1891-12	62	21	83	47	130	22	599	137	736
14	1902-01	1906-02	1895-11	1905-07	74	42	116	15	131	8	641	289	930
15	1913-07	1917-08	1906-10	1916-10	81	39	120	42	150	32	721	297	1018
16	1923-08	1929-04	1920-04	1926-07	40	35	75	50	125	17	347	187	534
17	1933-09	1937-04	1930-09	1935-07	36	22	58	76	134	55	313	255	568
18	1944-02	1947-05	1941-11	1945-09	27	19	46	63	109	43	141	128	269
19	1954-04	1958-03	1950-12	1955-10	40	18	58	73	131	44	251	195	446
20	1964-10	1968-11	1961-11	1966-05	35	19	54	86	140	68	133	94	227
21	1976-03	1979-12	1973-07	1977-07	32	16	48	76	124	47	182	91	273
22	1986-09	1989-11	1983-11	1987-07	34	10	44	81	125	53	205	68	273
23	1996-08	2001-11	1994-04	1998-01	28	17	45	72	117	26	171	138	309
24	2008-12	2014-04	2004-01	2011-01	59	25	84	42	126	3	509	307	816
				mean	50.6	26.4	77.0	55.1	131.3	31.2	367.7	198.1	565.7
				sd	19.5	12.3	29.3	22.4	11.2	18.8	220.6	87.4	284.2
25	–	–	2014-07	–	29+	–	29+	–	29+	–	27+	–	27+

Note: t_1 = elapsed time in months from FSD (cycle *n*) to Em (cycle *n*).
t_2 = elapsed time in months from Em (cycle *n*) to LSD (cycle *n*).
t_3 = elapsed time in months from FSD (cycle *n*) to LSD (cycle *n*).
t_4 = elapsed time in months from LSD (cycle *n*) to FSD (cycle *n* + 1).
t_5 = elapsed time in months from FSD (cycle *n*) to FSD (cycle *n* + 1).
t_6 = elapsed time in months from EM (cycle *n*) to FSD (cycle *n* + 1).
NSD (t_1) = number of spotless days in t_1.
NSD (t_2) = number of spotless days in t_2.
NSD (t_3) = number of spotless days in t_3.

Figure 5 displays the cyclic variation of (a) t_1, (b) t_2, and (c) t_3. Concerning t_1, runs-testing (Langley 1971) suggests that the distribution of t_1 values is nonrandom at the 5% level of significance. Hence, one can divide the t_1 values into the following two groups: (1) those of longer t_1 values (i.e., long lead times between FSD and Em) and (2) those of shorter t_1 values (i.e., short lead times between FSD and Em). SC10–SC15 represent the longer lead time SCs (depicted by the open circles), and SC16–SC23 are the shorter lead time SCs (depicted by the filled circles). Longer lead time SCs average t_1 = 71.3 months, with *sd* = 7.8 months, while shorter lead time SCs average t_1 = 35.3 months, with *sd* = 4 months. The t_1 value for SC24 (depicted as an open square and equal to 59 months) appears to more closely resemble that of the longer lead time group. Certainly, its value falls within the 90% prediction interval of longer lead time SCs (71.3 ± 15.7 months) but not within the 90% prediction interval of shorter lead time SCs (35.3 ± 7.6 months). Therefore, it seems more likely that SC24 is best described as being a longer lead time SC rather than a shorter lead time SC. Furthermore, assuming that SC24 marks the beginning of a new string of longer lead time SCs (like SC10–SC15), one suspects that SC25's t_1 value might also be representative of the longer lead time SCs, suggesting that the Em for SC25 should not be expected until about April 2020 (i.e., July 2014 + 70 months). If true, then SC24's cycle length (i.e., Em cycle 24 to Em cycle 25) will be about 148 months, the same cycle length as was observed for SC23. (Previously, Zięba and Nieckarz (2014) have called SC10–SC15 'passive cycles' and SC16–SC23 'active cycles.')

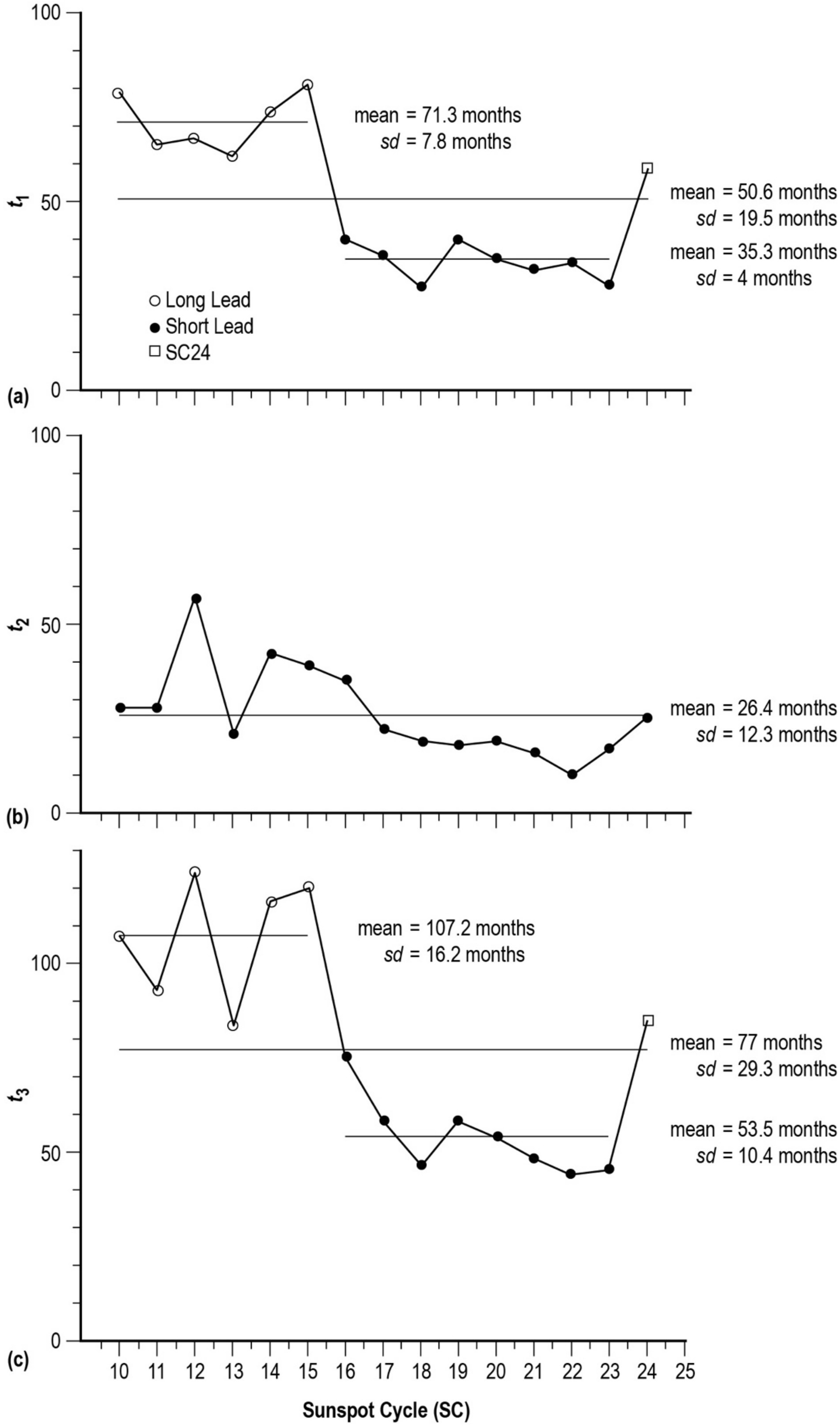

Figure 5. The cyclic variation of (a) t_1 (elapsed time in months from first spotless day (FSD) cycle *n* to epochs of minimum (Em) (cycle *n*)); (b) elapsed time in months (Continued)

from Em to the last spotless day (LSD); (t_2) and (c) elapsed time in months from FSD (cycle n) to FSD (cycle $n + 1$) (t_3). The overall means and standard deviations (*sd*) are given for each. Additionally, because runs-testing suggests that the distributions of t_1 and t_3 are nonrandom at the 5% level of significance, means and *sd* are also shown for the long lead (SC10–SC15) and short lead cycles (SC16–SC23). SC24's t_1 and t_3 values appear to resemble the t_1 and t_3 values for the long lead SCs.

Concerning t_2 (the elapsed time in months from Em for cycle n to LSD for cycle n), on average, its length is 26.4 months, with $sd = 12.3$ months. Presuming SC25's t_2 to be equal to 26.4 months, one surmises that SC25's LSD would not occur until about June 2022 (i.e., April 2020 + 26 months). (The 90% prediction interval for t_2 is 26.3 ± 21.7 months.)

Concerning t_3, like t_1, runs-testing suggests that its distribution is nonrandom at the 5% level of significance. Hence, t_3 can be divided into two groups as before with t_1. Presuming that SC24's t_3 is representative of the longer duration group (like SC10–SC15) and that it represents the start of another string of longer duration t_3 SCs, one suspects SC25's t_3 to be about 104 months in length, or that LSD for SC25 should not be expected until about July 2014 + 104 months, or about March 2023. If true, then this seems to suggest that SC25's t_2 will be longer than the 26-month average length (instead being about 45 months).

Figure 6 displays the cyclic variation of (a) t_4, (b) t_5, and (c) t_6. Concerning t_4, like t_1 and t_3, runs-testing suggests that its distribution is nonrandom at the 5% level of significance. The t_4 value for SC24 (depicted as an open square and equal to 42 months) appears to more closely resemble that of the t_4 values of SC10–SC15. Presuming that SC24 indeed marks the beginning of a new string of shorter t_4 cycles (like SC10–SC15), one expects SC25's t_4 value also to be representative of the shorter SCs, suggesting that the FSD for SC26 should not be expected until about 36 months following LSD for SC25 (or about March 2026 based on the LSD for SC25 being March 2023 from above).

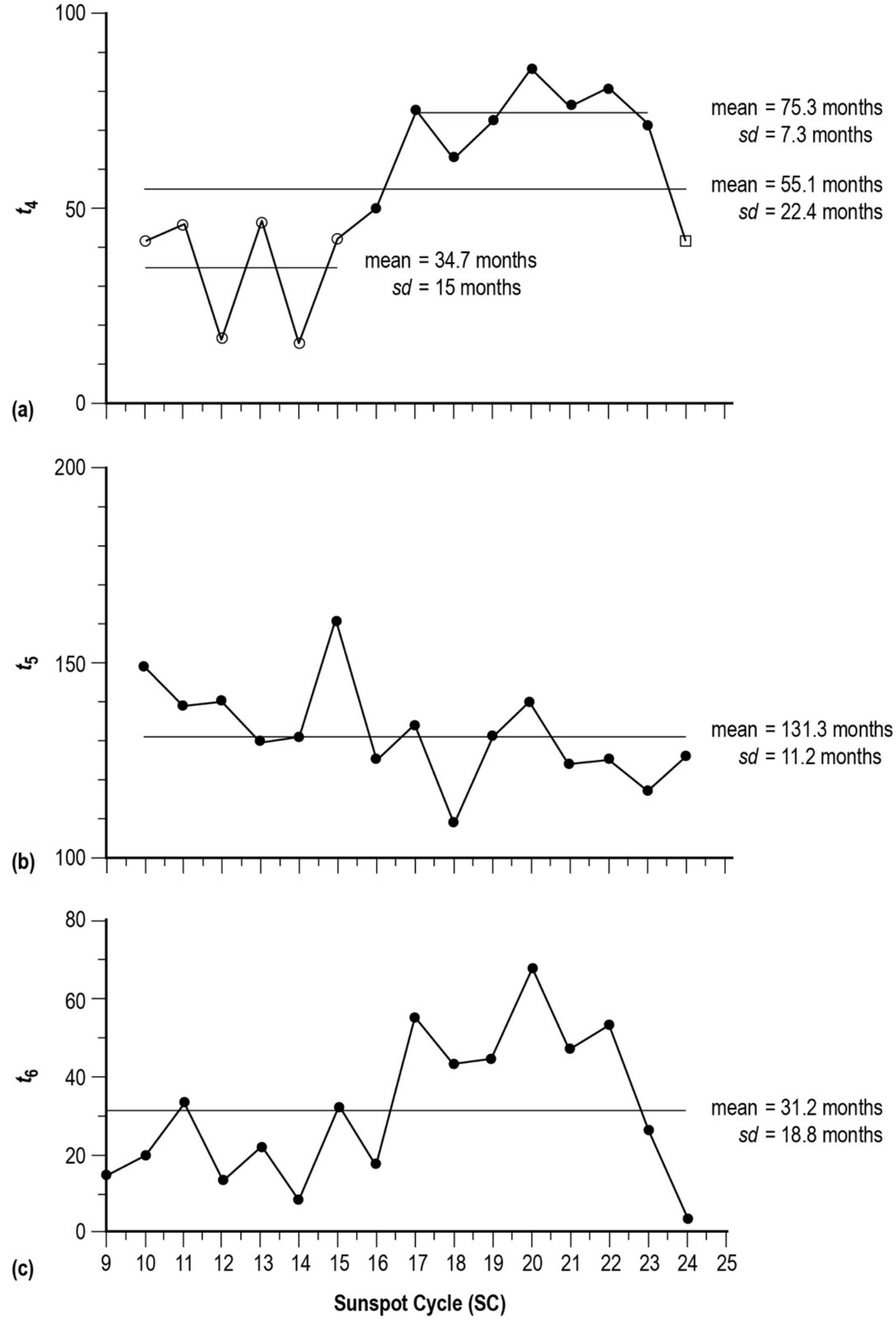

Figure 6. The cyclic variation of (a) t_4, (b) t_5, and (c) t_6. The overall means and standard deviations (*sd*) are given for each. Additionally, because runs-testing suggests that the distribution of t_4 is nonrandom at the 5% level of significance, the mean and *sd* for SC10–SC15 and SC17–SC23 are also shown. SC24's t_4 value appears to resemble the t_4 value for SC10–SC15.

Concerning t_5, its mean is 131.3 months, with sd = 11.2 months. Since FSD for SC25 is known to be July 2014, one doesn't expect FSD for SC26 to occur until about June 2025. (From above, FSD for SC26 is estimated to occur about March 2026, which, if true, suggests that SC25's t_5 would equal about 140 months. The 90% prediction interval for t_5 is 131.3 ± 19.7 months.)

Concerning t_6 (the elapsed time in months from EM cycle n to FSD cycle $n + 1$), on average, it measures about 31.2 months, with sd = 18.8 months. For SC24 it measured only 3 months, the shortest t_6 value in the modern era of sunspot observations. Noticeable is that 10 of the 15 SCs have had $t_6 \leq 33$ months, spanning 3–33 months and averaging about 18.9 months (sd = 9.8 months). Only SC17–SC22 had $t_6 > 33$ months, spanning 43–68 months and averaging about 51.7 months (sd = 9.3 months). Presuming SC25 will have t_6 = 19 months, one expects EM for SC25 to occur about August 2024, based on the presumed occurrence of SC26's FSD in March 2026 (from the discussion above for t_4), or to occur about November 2023, based on the presumed occurrence of SC26's FSD in June 2025 (from the discussion above for t_5).

Figure 7 shows the cyclic variation of (a) NSD (t_1), (b) NSD (t_2), and (c) NSD (t_3). Concerning NSD (t_1), on average, it measures about 367.7 days, with sd = 220.6 days. NSD (t_1) values are found to be well above the long-term mean for SC12–SC15 and SC24. For these SCs, they have an average NSD (t_1) = 647.4, with sd = 101.6, whereas for SC10–SC11 and SC16–SC23 the average NSD (t_1) = 227.8, with sd = 77.1. (It should be noted that, through January 2017, SC25 has NSD (t_1) = 37. Obviously, NSD (t_1) for SC25 will continue to increase as time goes on, exceeding about 400 if SC25 is similar to SC12–SC15 and SC24, or being less than about 370 if SC25 is more like SC10–SC11 and SC16–SC23, where these limits represent the 95% prediction interval extremes (lower and upper, respectively) for the two groupings.)

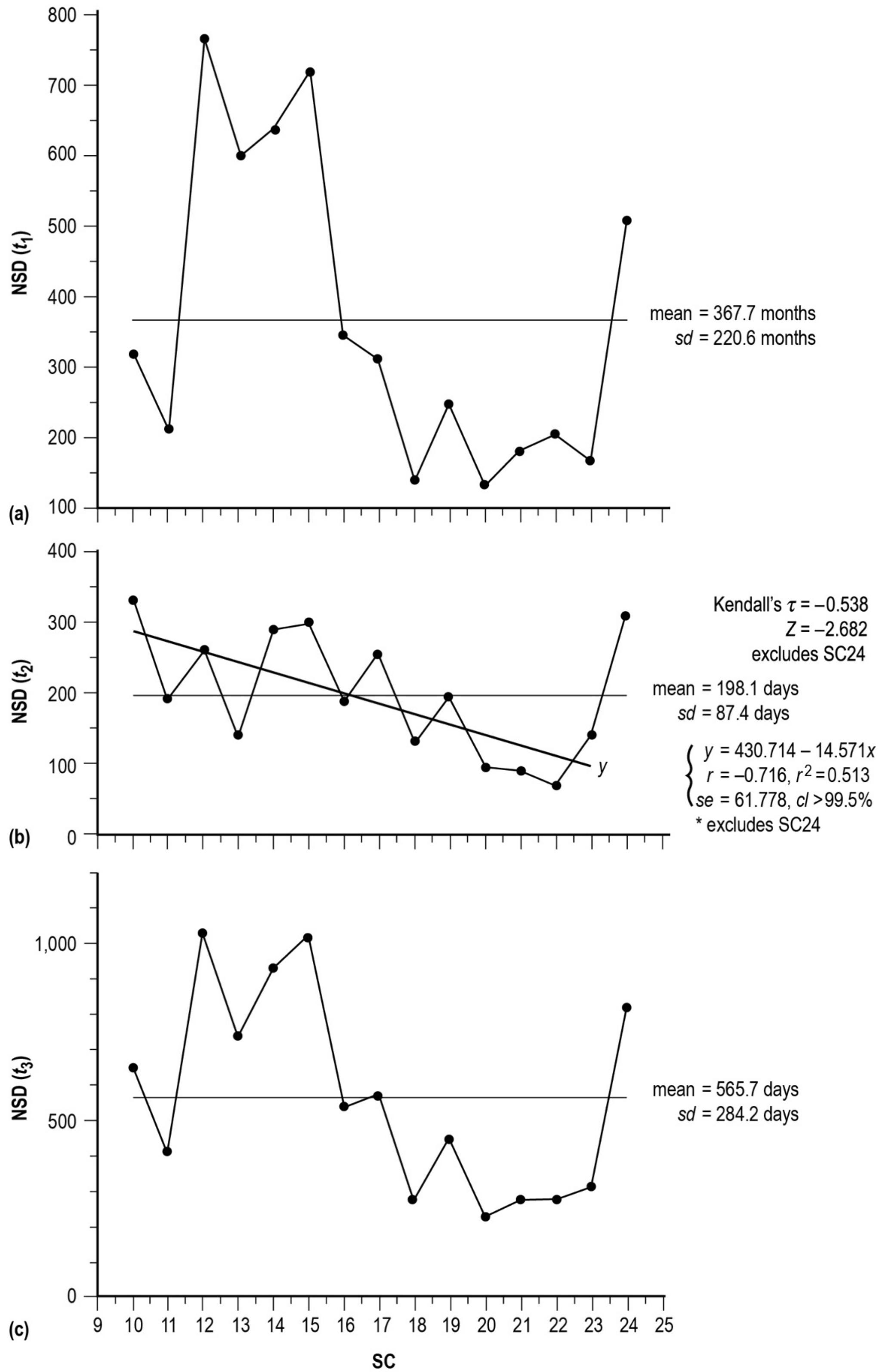

Figure 7. The cyclic variation of (a) the number of spotless days (NSD) during t_1 (NSD (t_1)), (b) the NSD during (t_2) (NSD (t_2)), and (c) the NSD during t_3 (NSD (t_3)). (Continued)

The overall means and standard deviation *sd* are given for each. Additionally, a statistically significant downward trend is noted for NSD (t_2) for SC10–SC23.

Concerning NSD (t_2), on average, it measures 198.1 days, with sd = 87.4 days. Interesting, however, is the apparent downward trend in NSD (t_2) values between SC10 and SC23. Based on linear regression analysis, the inferred relationship is described as $y = 430.714 - 14.571x$, where y is NSD (t_2), and x is SC number. The inferred regression has $r = -0.716$, $r^2 = 0.513$, $se = 61.778$, and $cl > 99.5\%$. Based on Kendall's $\tau = -0.588$ and $Z = -2.682$, one infers that the association between NSD (t_2) and SC is statistically important at the 2% level of significance. It is apparent, however, that SC24's NSD (t_2) value fails to adhere with the inferred trend (i.e., it is a statistical outlier), being about 3.7 se greater than what the inferred regression predicts (307 observed versus 81 from the inferred regression).

Concerning NSD (t_3), on average it measures 565.7 days, with sd = 284.2 days, having a cyclic behavior that mimics that of NSD (t_1). For SC12–SC15 and SC24, NSD (t_3) averages about 905.4 days, with sd = 127.2 days, while for SC10–SC11 and SC16–SC23, it averages about 395.9 days, with sd = 149.4 days.

Figure 8 depicts the scatter plot of NSD (t_3) versus NSD (t_1). Clearly, a very strong linear correlation exists between the two parameters, one described as $y = 105.711 + 1.251x$, where y is NSD (t_3) and x is NSD (t_1). The inferred linear regression has $r = 0.971$, $r^2 = 0.944$, $se = 70.491$, and cl » 99.9%. Based on Kendall's $\tau_b = 0.900$ and $Z = 4.674$ (one uses Kendall's τ_b because there was one tie), the association is inferred to be highly statistically important ($cl > 99.9\%$). Based on Fisher's exact test for the 2×2 contingency table, one finds the probability P of obtaining the observed distribution, or one more suggestive of a departure from independence, to be $P = 0.00016$. Hence, given the NSD (t_1), one can estimate the total NSD that are expected to occur in the vicinity bounding Em for a SC (i.e., between FSD and LSD).

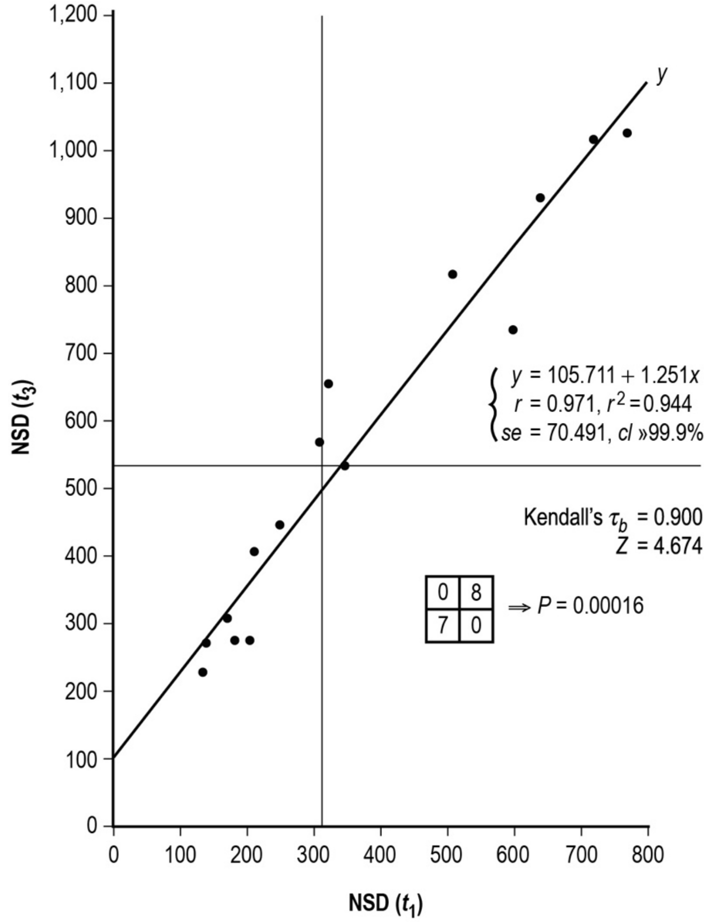

Figure 8. Scatterplot of number of spotless days (NSD) during t_3 (NSD (t_3)) versus the NSD during t_1 (NSD (t_1)). The results of linear regression analysis and nonparametric analyses are given.

Figure 9 plots (a) the mean and greatest observed (GO) monthly NSD value (based on SC10–SC24) relative to Em for elapsed time in months from t = –36 to t = +24 and (b) the number of SCs having monthly NSD >10 days (thin line) and monthly NSD >20 days (thick line) for the same interval t = –36 to t =+24. Clearly, at t = –36, only 2 of the 15 SCs (SC12 and SC13) had a monthly count of NSD >10 and none had NSD >20. It is not until t = –14 before the majority of SCs (8 of 15) had NSD >10 and not until t = –4 before monthly NSD >20 was observed for the majority of SCs. As yet (i.e., through January 2017), there has not been an occurrence of monthly NSD >10 for SC25. So, it seems likely that SC25's Em remains more than 36 months away, indicating Em for SC25 probably after January 2020. For convenience, Table 5 is included to provide monthly counts of NSD for SC10–SC24, the mean and *sd*, LO and GO monthly NSD values, and the number of SCs having monthly NSD >10 and >20.

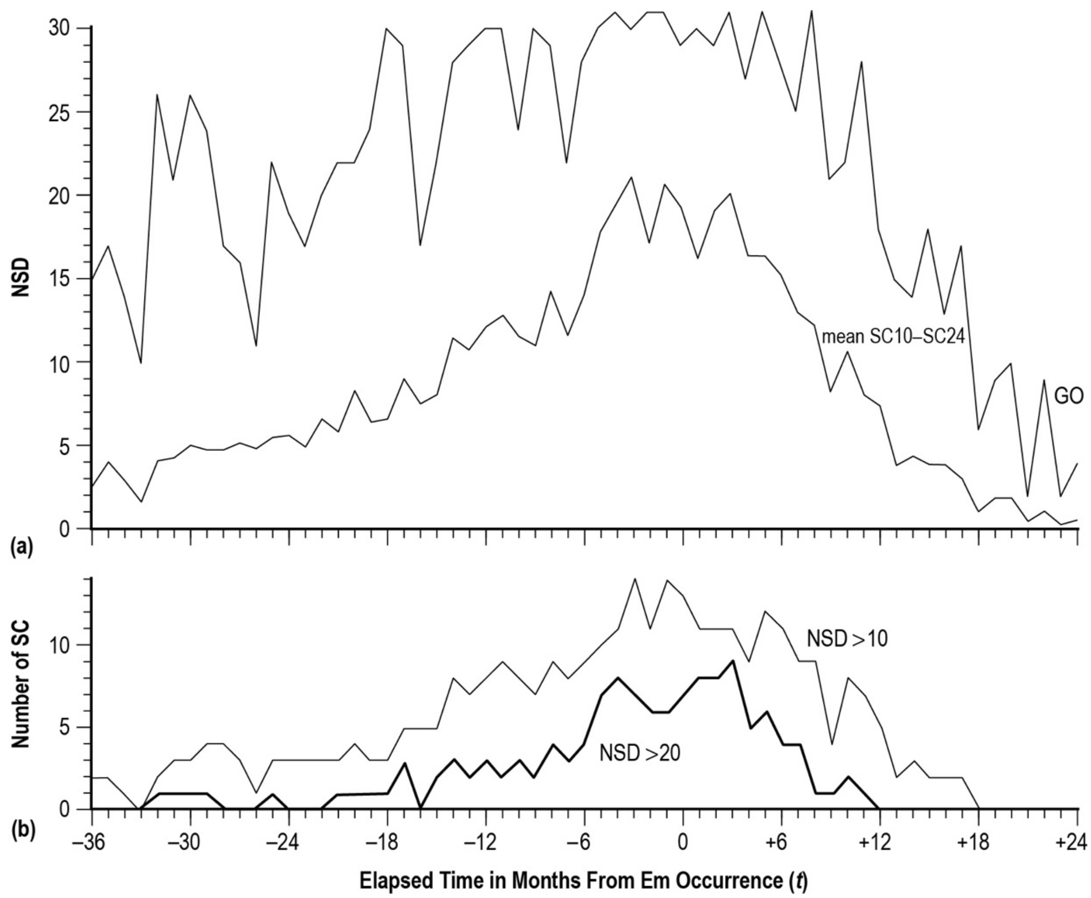

Figure 9. (a) The monthly variation of the number of spotless days (NSD) for elapsed time in months (*t*) relative to the epoch of sunspot minimum (Em) for *t* = –36 to +24 months, based on the mean of SC10–SC24 and the greatest observed (GO) values; (b) the number of sunspot cycles (SCs) having NSD >10 days per month (thin line) and >20 days per month (thick line).

Table 5. Monthly number of spotless days (NSD) for elapsed time from months (*t*) = –36 to *t* = +24 months relative to the epoch of sunspot minimum (SSN min) (Em) for sunspot cycles SC10–SC24 (Continued).

	SC																				
t	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	mean	sd	Least Observed	Greatest Observed	*n*>10	*n*>20
–36	0	0	15	15	2	5	1	1	0	0	0	0	0	0	0	2.6	5.2	0	15	2	0
–35	0	3	17	11	10	10	4	0	0	0	3	0	0	0	3	4.1	5.4	0	17	2	0
–34	0	0	9	5	4	4	0	1	0	0	3	0	4	0	14	2.9	4.1	0	14	1	0
–33	2	0	8	0	1	1	0	1	0	0	1	0	0	0	10	1.6	3.1	0	10	0	0
–32	1	0	26	4	7	19	0	4	0	0	0	0	0	0	0	4.1	7.9	0	26	2	1

Table 5. Monthly number of spotless days (NSD) for elapsed time from months (*t*) = –36 to *t* = +24 months relative to the epoch of sunspot minimum (SSN min) (Em) for sunspot cycles SC10–SC24 (Continued).

	SC																				
t	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	mean	sd	Least Observed	Greatest Observed	*n*>10	*n*>20
–31	0	2	21	11	2	16	0	1	0	0	0	0	0	0	4	4.2	6.6	0	21	3	1
–30	1	1	26	12	9	19	0	0	0	0	0	0	0	0	5	5.0	8.1	0	26	3	1
–29	1	0	11	11	24	13	3	0	0	0	0	0	0	0	2	4.7	7.1	0	24	4	1
–28	0	0	17	13	11	14	1	0	0	0	0	6	0	5	4	4.7	6.1	0	17	4	0
–27	1	1	14	4	16	5	6	11	3	0	1	7	0	6	2	5.1	5.0	0	16	3	0
–26	0	0	7	7	10	11	1	3	2	6	3	7	0	5	10	4.8	3.8	0	11	1	0
–25	0	0	15	12	8	22	0	10	1	9	0	2	0	0	3	5.5	6.9	0	22	3	1
–24	0	2	19	12	10	19	5	0	0	0	0	1	8	0	8	5.6	6.8	0	19	3	0
–23	6	2	7	11	13	17	4	5	1	0	1	0	4	2	0	4.9	5.2	0	17	3	0
–22	8	2	8	17	14	20	7	7	0	0	4	5	1	0	6	6.6	6.2	0	20	3	0
–21	7	2	15	10	3	22	9	2	2	0	0	0	0	0	15	5.8	7.0	0	22	3	1
–20	4	2	11	16	8	15	10	7	6	0	1	0	11	1	22	8.3	7.1	0	22	4	1
–19	8	1	2	20	9	24	10	11	5	2	2	0	0	0	4	6.5	7.3	0	24	3	1
–18	1	7	4	6	14	30	4	8	2	2	6	1	5	0	11	6.7	7.5	0	30	3	1
–17	5	10	16	21	21	29	3	12	1	2	0	0	7	1	9	9.1	9.0	0	29	5	3
–16	4	4	17	7	13	17	13	5	3	2	0	0	0	13	9	7.6	5.9	0	17	5	0
–15	6	12	4	11	8	21	14	1	3	7	0	4	1	7	22	8.1	6.8	0	22	5	2
–14	16	0	23	28	16	16	14	10	0	17	1	2	0	2	28	11.5	10.2	0	28	8	3
–13	0	1	10	15	29	16	13	14	4	11	3	9	7	6	24	10.8	8.1	0	29	7	2
–12	5	0	23	15	30	22	17	16	2	8	2	9	16	5	13	12.2	8.8	0	30	8	3
–11	17	2	22	17	20	30	12	6	1	8	1	18	14	7	19	12.9	8.6	1	30	9	2
–10	9	6	20	24	23	8	14	12	0	0	5	13	11	7	22	11.6	7.8	0	24	8	3
–9	6	3	15	17	30	21	9	6	0	14	1	9	11	7	17	11.1	8.1	0	30	7	2
–8	21	9	29	11	18	27	11	12	9	9	8	0	24	6	20	14.3	8.4	0	29	9	4
–7	19	5	22	8	15	17	21	12	7	1	2	0	4	13	21	11.7	7.4	0	22	8	3
–6	21	13	18	13	28	22	23	9	3	9	7	7	6	15	17	14.1	7.3	3	28	9	4
–5	30	5	30	20	27	19	21	22	13	25	4	9	5	10	28	17.9	9.5	4	30	10	7
–4	25	15	31	27	28	29	13	21	10	22	10	6	9	17	29	19.5	8.5	6	31	11	8
–3	30	27	18	17	21	27	19	14	10	30	20	14	26	17	26	21.1	6.3	10	30	14	7
–2	9	31	28	16	16	31	14	24	6	26	11	16	5	5	21	17.3	9.2	5	31	11	6
–1	21	26	18	25	31	30	20	30	20	14	18	18	8	14	17	20.7	6.6	8	31	14	6

Table 5. Monthly number of spotless days (NSD) for elapsed time from months (*t*) = –36 to *t* = +24 months relative to the epoch of sunspot minimum (SSN min) (Em) for sunspot cycles SC10–SC24 (Continued).

	SC																				
t	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	mean	sd	Least Observed	Greatest Observed	*n*>10	*n*>20
0	26	12	29	17	20	25	29	15	27	24	15	5	18	0	28	19.3	8.9	0	29	13	7
1	30	20	26	20	28	30	4	23	12	28	10	2	2	25	26	16.4	11.9	2	30	11	8
2	19	24	26	16	17	27	3	28	29	29	6	6	5	28	23	19.1	9.7	3	29	11	8
3	30	26	31	23	30	21	9	30	26	14	2	7	17	9	28	20.2	9.8	2	31	11	9
4	17	18	17	10	20	27	24	21	10	10	4	24	7	12	27	16.5	7.4	7	27	9	5
5	31	20	24	14	25	23	29	8	13	24	4	0	19	13	20	16.5	9.6	0	31	12	6
6	22	16	20	2	28	24	22	18	5	12	13	3	2	13	20	15.3	7.8	2	28	11	4
7	20	13	12	9	22	21	25	10	5	15	8	3	0	12	22	13.1	7.6	0	25	9	4
8	17	9	15	12	18	21	17	4	2	15	4	15	0	4	31	12.3	8.4	0	31	9	1
9	21	7	12	7	4	1	5	17	6	0	10	6	7	3	19	8.3	6.3	0	21	4	1
10	22	13	12	11	17	19	0	12	4	3	9	6	9	2	21	10.7	6.9	0	22	8	2
11	15	5	10	3	28	15	1	13	3	18	3	2	0	11	16	8.2	6.2	0	28	7	1
12	17	3	12	5	9	14	0	18	3	11	3	7	0	2	9	7.5	5.9	0	18	5	0
13	4	0	7	2	0	15	0	11	1	4	7	4	0	0	3	3.9	4.5	0	15	2	0
14	14	3	6	0	11	9	0	11	2	4	3	2	0	1	0	4.4	4.7	0	14	3	0
15	18	2	5	0	2	12	8	11	1	3	4	0	0	0	3	3.9	4.0	0	18	2	0
16	13	10	2	0	6	2	9	0	0	1	0	4	0	0	12	3.9	4.8	0	13	2	0
17	0	0	4	1	6	0	17	0	1	1	2	0	0	3	12	3.1	5.0	0	17	2	0
18	6	1	0	0	0	0	2	4	4	3	0	0	0	0	2	1.1	1.5	0	6	0	0
19	6	0	5	0	0	1	6	9	1	0	1	0	0	0	0	1.9	3.0	0	9	0	0
20	9	0	0	0	10	0	0	6	0	0	0	0	0	0	3	1.9	3.5	0	10	0	0
21	0	0	0	2	0	2	0	0	0	0	0	0	0	0	2	0.4	0.8	0	2	0	0
22	0	0	0	0	0	9	2	1	0	0	1	0	0	0	3	1.1	2.4	0	9	0	0
23	0	0	2	0	1	0	1	0	0	0	0	0	0	0	0	0.3	0.6	0	2	0	0
24	0	0	1	0	1	0	1	0	0	0	0	0	0	0	4	0.5	1.1	0	4	0	0

In conclusion, based on annual averages, NSD max has always occurred at SSN min for SC10–SC24, inferring a very strong inverse linear correlation of the form $y = 24.536 - 0.068x$ and having $r = -0.925$ and $se = 1.844$ units of SSN. Hence, a large NSD max during SSN min indicates a smaller SSN min, while a small NSD max indicates a larger SSN min. Comparisons of NSD relative to Em and EM suggest that SC24's NSD behavior is quite similar to that experienced in SC12 and SC14, both in terms of size of NSD and timing. Hence, this seems to suggest that SC24 will be a long-period SC, having a minimum to minimum period of about 12 years. The FSD for SC25 occurred in July 2014, just 3 months following SC24's EM, an interval shorter than was observed for SC12 (13 months) and SC14 (8 months). Through December 2016, some 27 spotless days have been reported (an additional 10 have been reported in January 2017). The NSD occurring during the decline of SC24 (which are associated with the approaching SC25's minimum) will continue to increase over time. At present, it appears that SC25's SSN min remains more than 36 months away, since, as yet, there has been no occurrence of a monthly NSD >10 days per month.

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