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SIMPLE METHODS FOR PREDICTING THE SIZE AND TIMING OF SUNSPOT CYCLE 25: ADDITIONAL REMARKS

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ABSTRACT

A simple method based on the number of continuous months bounding sunspot minimum occurrence with smoothed monthly mean sunspot number R <20 (i.e., N(R<20)) is found to be useful for predicting the size and timing of a sunspot cycle (SC). In particular, an SC having N(R<20) <19 months tends to have a larger sunspot minimum (Rm) and maximum (RM) amplitude and a shorter ascent (ASC) and period (PER), while an SC having N(R<20) ≥19 months tends to have a smaller Rm and RM and a longer ASC and PER. SC25, the present ongoing cycle, had N(R<20) = 43 months, suggesting Rm = 5.6 ± 4.6, RM = 144.2 ± 43.5, ASC = 59 ± 14 months and PER = 132 ± 14 months. Instead, based on inferred regression equations and using N(R<20) = 43 months, SC25 is expected to have Rm = 3.6 ± 2.8, RM = 130.9 ± 39.7, ASC = 62 ± 11 months and PER = 137 ± 14 months. For SC25, Rm = 1.8 occurred in December 2019 and R exceeded 116.4 (SC24's RM) in February 2023. Therefore, SC25's RM will be larger than that observed for SC24 and not smaller. For SC25, RM = 148.5 ± 21.1 is the projected value based on the average of several techniques for estimating RM. Such a value means the 2-cycle moving average for SC24 will be 140.4, some 32 units of sunspot number below that observed for SC23, further suggesting that SC24, indeed, marks the beginning of another three to five cycles of extended intervals of low sunspot number minimum- and maximum-amplitude cycles.

INTRODUCTION

Sunspot cycle (SC) 25 continues growing in amplitude (i.e., smoothed monthly mean sunspot number R), having surpassed the maximum amplitude (RM) of SC24 (116.4) in February 2023 (117.9). Indeed, the early behavior of SC25 is strongly suggestive that it is a slow-rising-long-period sunspot cycle (i.e., one having an ascent duration, ASC, equal to 49 months or longer and period, PER, equal to 133 months or longer) with maximum amplitude occurrence expected on or after January 2024 (Wilson 2022). Prior to its onset, speculation suggested that SC25 likely would be a relatively small cycle with maximum amplitude similar to that of SC24, or smaller (cf. https://en.wikipedia.org/wiki/Solar_cycle_25), and that, perhaps, this would be an indication of the imminent occurrence of another Maunder–like or Dalton-like minimum (i.e., an extended period of low sunspot number spanning several decades; cf. Hoyt and Schatten 1996; Russell, Luhmann and Jian 2010; Feynman and Ruzmaikin 2011; Zolotova and Ponyavin 2014; Zachilas and Gkana 2015; Usoskin, Arlt, Asvestari et. al. 2015; Javaraiah 2017; Singh and Bhargawa 2019).

In this study, the lengths (i.e., number of months) of the continuous intervals bounding Rm (minimum amplitude) having R <20 (i.e., N(R<20)) are determined for SC00-25 and linear regression analysis is performed between Rm, RM, ASC and PER against N(R<20). The inferred correlative relationships are then used to predict, in particular, Rm, RM, ASC and PER for SC25. Also examined is the likelihood that SC24-25, and possibly SC26 and beyond, represents a recurrence of another Dalton-like minimum, a reflection of the Centennial Gleissberg Cycle (Gleissberg 1965; Feynman and Fougere 1984; Feynman and Ruzmaikin 2011).

METHODS AND MATERIALS

Smoothed monthly mean sunspot number R is taken from http://sidc.oma.be/silso/datafiles to determine N(R<20), Rm, RM, ASC and PER for SC00-25. 2-cycle moving averages (2-cma) are employed to show trends in the cyclic values, where the 2-cma is representative of the variation of the Hale cycle (i.e., two consecutive sunspot cycles). Recall that the Sun's magnetic cycle spans two consecutive sunspot cycles, with the northern hemisphere displaying positive-leading polarity of sunspots and the southern hemisphere displaying negative-leading polarity in odd-numbered sunspot cycles, being reversed in even-numbered sunspot cycles (Howard 1977).

RESULTS AND DISCUSSION

Table 1 gives the cyclic values of N(R<20), Rm, RM, ASC and PER for SC00-25. Also given are the means, standard deviations (sd) and medians (med), both for the entire grouping of SC00-25 and for the two subgroupings based on the median value of N(R<20) = 19 (i.e., those having N(R<20) less than 19 months and those having N(R<20) greater than or equal to 19 months), and the results of runs testing for randomness (Lapin 1978). Of the various parameters, only RM is found to be non-randomly distributed, having a normal deviate $z = -2.17$. R <20 was chosen as the differentiating criterion because R = 20 is a value slightly larger than the largest Rm value occurring in SC00-25 (18.6 for SC02) and is a value larger than that believed to have been experienced during the Maunder minimum (cf. Wilson 1988; Beer, Tobias and Weiss 1998; Hathaway and Wilson 2004; Kovaltsov, Usoskin and Mursula 2004; Hathaway 2015; Usoskin 2017.)

Table 1. N(R < 20), Rm, RM, ASC and PER for SC00-25

SC	N(R<20)	Rm	RM	ASC	PER
00	–	–	158.9	–	–
01	21	14	144.1	75	135
02	3	18.6	193	39	108
03	15	12	264.3	35	111
04	11	15.9	235.3	41	163
05	44	5.3	82	82	147
06	72	0	81.2	70	154
07	50	0/2	119.2	78	126
08	16	12.2	244.9	40	116
09	4	17.6	219.9	55	149
10	22	6	186.2	50	135
11	13	9.9	234	41	141
12	27	3.7	124.4	60	135
13	35	8.3	146.5	46	142
14	42	4.5	107.1	49	138
15	42	2.5	175.7	49	121
16	18	9.4	130.2	56	121
17	24	5.8	198.6	43	125
18	11	12.9	218.7	39	122
19	18	5.1	285	47	126
20	13	14.3	156.6	49	137
21	6	17.8	232.9	45	126
22	19	13.5	212.5	38	119
23	19	11.2	180.3	63	148
24	39	2.2	116.4	64	132
25	43	1.8	–	–	–
Mean	25	9	177.7	52	132
sd	17	5.8	56.8	14	14
Median	19	9.4	175.7	49	133.5
na	14	13	13	13	12
nb	11	12	12	11	12
Ra	6	6	4	7	5
z	–0.57	–.06	–2.17	0.4	–1.2
N(R<20) <19 months					
Mean	11.6	13.2	219.5	44.3	129.1
sd	5.3	4.1	45	6.8	16.8
n	11	11	11	11	11
N(R<20) ≥19 months					
Mean	35.6	5.6	144.2	59	135.2
sd	15	4.6	43.5	14.3	10.7
n	14	14	13	13	13
t	–5.05	4.3	4.16	–3.12	–1.08

Notes:

SC means sunspot cycle
N(R<20) is the number of contiguous months bounding Rm with R<20
Rm is sunspot minimum amplitude using smoothed monthly mean sunspot number R
RM is sunspot maximum amplitude using smoothed monthly mean sunspot number R
ASC is the ascent period in months from Rm occurrence to RM occurrence
PER is the period or length of SC in months from Rm occurrence SCn to Rm occurrence SCn+1
na is the number of entries above the median
nb is the number of entries below the median
Ra is the number of runs of na
z is the normal deviate for the sample
sd is the standard deviation
n is the number of entries
t is the t statistic for independent samples

Similarly, Table 2 gives the mean, sd and median for the entire grouping (SC01-24) and the two subgroups based on the median value of N(R<20) = 20.3, as well as the results of runs testing for randomness but now using 2-cma values.

Table 2. 2-cma of N(R < 20), Rm, RM, ASC and PER for SC01-24

SC	N(R<20)	Rm	RM	ASC	PER
01	–	–	160	–	–
02	10.5	15.8	198.6	47	115.5
03	11	14.6	239.2	37.5	123.3
04	20.3	12.3	204.2	49.8	146
05	42.8	6.6	120.1	68.8	152.8
06	59.5	1.4	90.9	75	145.3
07	47	3.2	141.1	66.4	130.5
08	21.5	10.6	207.2	53.3	126.8
09	11.5	13.4	217.7	50	137.3
10	15.3	9.9	206.6	49	140
11	18.8	7.4	194.7	48	138
12	25.5	6.4	157.3	51.8	138.3
13	34.8	6.2	131.1	50.3	139.3
14	40.3	5	134.1	D48.3	134.8
15	36	4.7	147.2	50.8	125.3
16	25.5	6.8	158.7	51	122
17	19.3	8.5	186.5	45.3	123.3
18	16	9.2	230.3	42	123.8
19	15	9.4	236.3	45.5	127.8
20	12.5	12.9	207.8	47.5	131.5
21	11	15.9	208.7	44.3	127
22	15.8	14	209.6	46	128
23	24	14.5	172.4	57	136.8
24	35	4.4	–	–	–
25	–	–	–	–	–
Mean	24.7	9.8	180.9	51.1	132.4
sd	13.4	4	40.3	8.8	9.2
Median	20.3	9.2	194.7	49.4	131
na	12	12	12	11	11
nb	11	11	11	11	11
Ra	3	3	3	4	4
z	–2.67	–2.67	–2.67	–1.67	–1.67
N(R<20) <20.3					
Mean	14.2	11.9	212.4	45.6	128.7
sd	3.1	3.1	17	3.5	7.5
n	11	11	11	11	11
N(R<20) ≥20.3					
Mean	34.4	6.8	151.3	56.6	136.2
sd	11.8	3.8	34.5	9.2	9.6
n	12	12	11	11	11
t	–5.5	3.51	5.27	–3.71	–2.04

Notes:
SC means sunspot cycle
N(R<20) is the number of contiguous months bounding Rm with R<20
Rm is sunspot minimum amplitude using smoothed monthly mean sunspot number R
RM is sunspot maximum amplitude using smoothed monthly mean sunspot number R
ASC is the ascent period in months from Rm occurrence to RM occurrence
PER is the period or length of SC in months from Rm occurrence SCn to Rm occurrence SCn+1
sd is the standard deviation
na is the number of entries above the median
nb is the number of entries below the median
Ra is the number of runs of na
z is the normal deviate for the sample
n is the number of entries
t is the t statistic for independent samples

From Table 1, one finds that the two subgroupings based on N(R<20) have means that are statistically independent at $\alpha = 0.05$ or higher level of statistical significance for all parameters, except PER. Hence, if one knows the value of N(R<20), one can simply estimate each of the parameters for that particular cycle. Because N(R<20) = 43 for SC25, one predicts SC25 to have Rm = 5.6 ± 4.6, RM = 144.2 ± 43.5, ASC = 59 ± 14 months and PER = 132 ± 14 months. (For SC25, Rm = 1.8 was observed in December 2019.)

From Table 2, one finds that the variance using 2-cma for each of the parameters is greatly reduced (50% or more), as compared to using the observed cyclic values. For all parameters the t statistic for independent samples is statistically significant at $\alpha = 0.05$ or higher.

Figures 1 and 2 display the cyclic variation (thin line) and 2-cma (thick line) of N(R<20), Rm, RM, ASC and PER. All parameters show large variations, both above and below their respective median values, with each variation lasting typically 3 or more consecutive cycles, easily discerned using the 2-cma values. For example, Figure 1(a) shows the variation in N(R<20). Discernable are large variations above R = 19 between SC05-07, SC13-15 and what appears to be another one beginning with SC24. These periods of larger than median value have previously been associated with extended periods of reduced sunspot number associated with the Dalton minimum (SC05-07) and the minimum near the beginning of the 20^{th} Century (SC13-15). Plainly, large values of N(R<20) are associated with smaller values of Rm (Figure 1(b)) and RM (Figure 1(c)), while small values of N(R<20) are associated with large values of Rm and RM (i.e., sunspot number amplitude varies inversely with N(R<20)). Such behavior is less apparent in ASC (Figure 2(a)) and PER (Figure 2(b)).

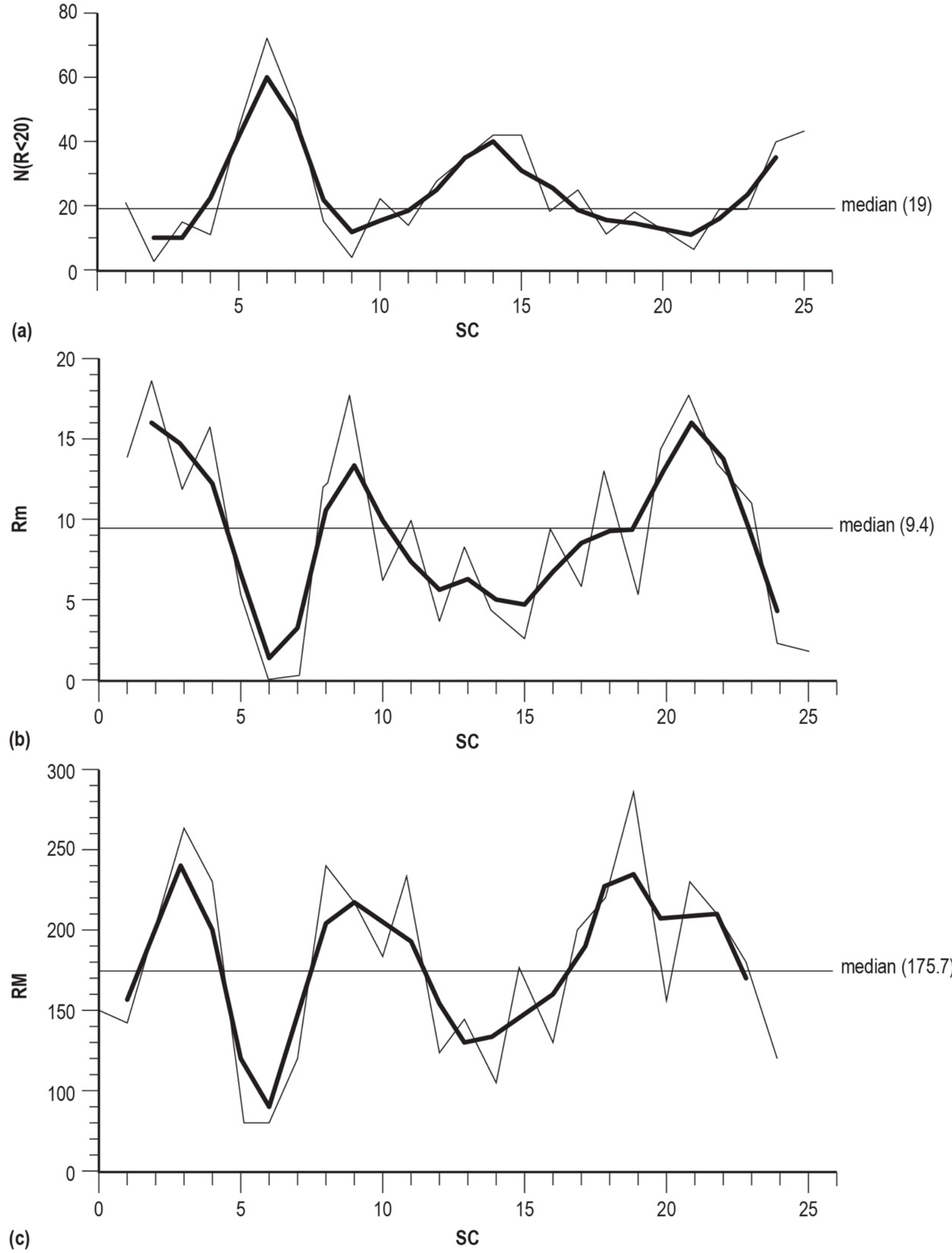
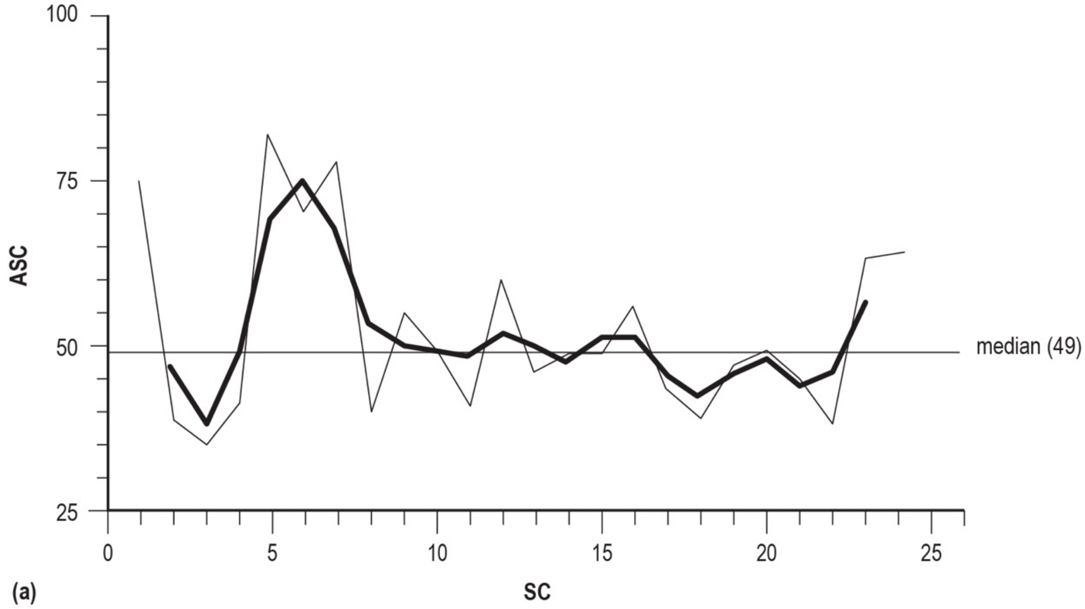


Figure 1. (a) The variation of N(R< 20) for SC00-25; (b) the variation of Rm for SC01-25; and (c) the variation of RM for SC00-24. The medians are shown (19, 9.4 and 175.7, respectively). The thin line is the actual cyclic value, and the thick line is the 2-cma.



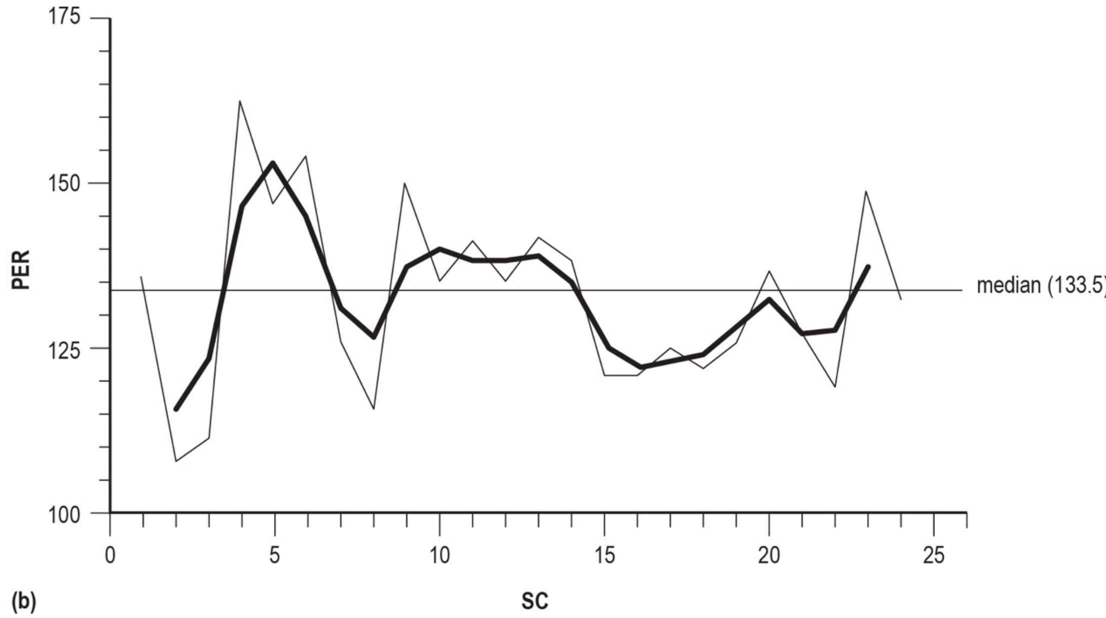


Figure 2. (a) The variation of ASC for SC01-24; and (b) the variation of PER for SC01-24. The medians are shown (49 and 133.5, respectively). The thin line is the actual cyclic value, and the thick line is the 2-cma.

Figure 3 depicts the scatterplots of Rm, RM, ASC and PER versus N(R<20). In each of the plots the inferred regression line is shown and various statistics are given, including the inferred regression equation *y*, the inferred correlation coefficient *r*, the inferred coefficient of determination r^2 (a measure of the amount of variance explained by the independent variable *x*), the inferred standard error of estimate S_{yx} and the inferred *t* statistic for evaluating the statistical significance of the slope in the regression equation. Also given is the result of Fisher's exact test for 2×2 contingency tables (determined using the median values of the parameters, the thin vertical and horizontal lines), where P_o is the probability of obtaining the observed result and P is the probability of obtaining, not only the observed 2×2 table, but also those that are more suggestive of a departure from independence (chance). The tiny downward pointing arrow in Figure 3(b), (c) and (d) at N(R<20) = 43 is the known value of N(R<20) for SC25.

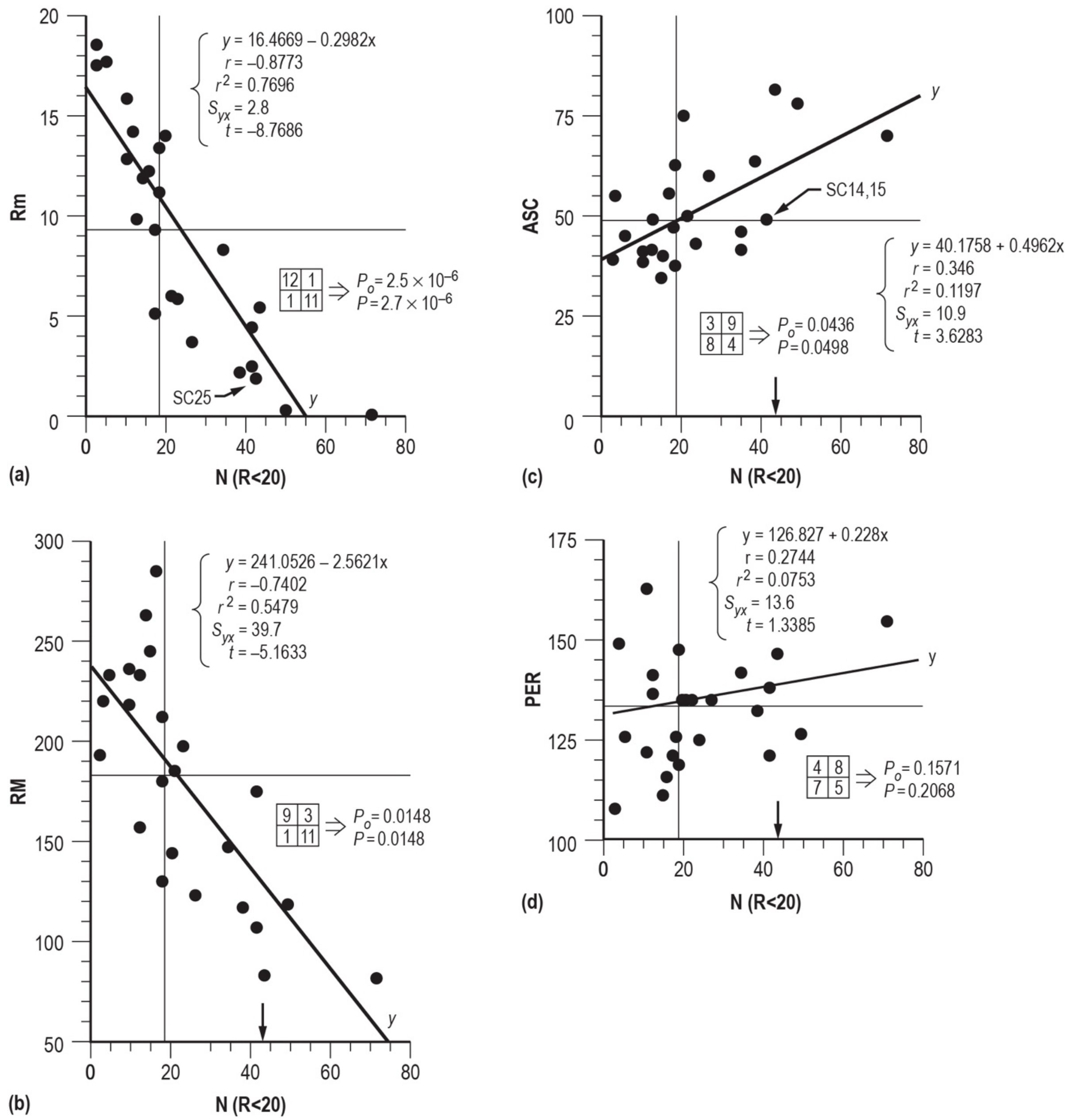


Figure 3. Scatterplots of (a) Rm versus N(<20); (b) RM versus N(R<20); (c) ASC versus N(R<20); and (d) PER versus N(R<20). Results of statistical analyses are given. The small arrow at N(R<20) = 43 is the value for SC25.

Figure 3(a) depicts the scatterplot of Rm versus N(R<20). A strong inverse relationship is shown (as expected). The larger (or smaller) the value of N(R<20), the smaller (or larger) the inferred Rm. The inferred regression equation has $r = -0.8773$ and $r^2 = 0.7696$, meaning that about 77% of the variance in Rm can be explained by the observed variation of N(R<20). Also, $S_{yx} = 2.8$ and $t = -8.7686$, inferring a highly statistically significant result. Because N(R<20) = 43 for SC25, one infers Rm = 3.6 ± 2.8 (the ±1 sigma prediction interval) for SC25. Based on Fisher's exact test, $P = 2.7 \times 10^{-6}$, inferring a highly statistically significant result. Hence, once N(R<20) >19, it became apparent that Rm would be <9.4 for SC25, which occurred nine months prior to SC25's observed Rm occurrence, being 1.8 in December 2019.

Figure 3(b) displays the scatterplot of RM versus N(R<20). Like Rm versus N(R<20), it too shows a statistically important inverse relationship to exist between RM and N(R<20). Because N(R<20) = 43 for SC25, one computes RM = 130.9 ± 39.7, inferring that RM for SC25 very probably will lie in the lower-right quadrant of the scatterplot.

Figure 3(c) shows the scatterplot of ASC versus N(R<20). Unlike the scatterplots of Rm versus N(R<20) and RM versus N(R<20), the scatterplot of ASC versus N(R<20) is positively correlated. This is because ASC is known to be inversely correlated against RM (i.e., the Waldmeier Effect; cf. Wilson 2015, 2019; Hathaway 2015). Because N(R<20) = 43 for SC25, one estimates ASC = 62 ± 11 months based on the statistically important inferred regression equation, suggesting that it likely will lie in the upper-right quadrant (i.e., ASC ≥49 months), inferring RM occurrence for SC25 on or after January 2024 (and prior to January 2026).

Figure 3(d) depicts the scatterplot of PER versus N(R<20). Of the four scatterplots, this one is the weakest. The scatterplot appears to be randomly distributed with only a slight tendency to associate longer PER with large N(R <20) and shorter PER with small N(R <20). Because N(R<20) = 43 for SC25, one estimates PER = 137 ± 14 months, or Rm occurrence for SC26 in May 2030 ± 14 months (prior to July 2031).

Because R for SC25 surpassed SC24's RM (116.4) in February 2023 (expected from the Even-Odd inferred relationship; Wilson 2015), it is now established that SC25 is not smaller in RM as compared to that of SC24, as often had been suggested. Based on the PER of SC24 (132 months), one projects SC25's RM = 181.4 ± 42.6 (cf. Wilson 2015, 2019). Similarly, based on SC25's Rm (1.8), one projects its RM = 136.5 ± 49.1 (Wilson 2015). Based on the minimum values of the geomagnetic indices in the vicinity of Rm (which occurred five months after the Rm occurrence), one projects SC25's RM = 157.6 ± 29.0 (based on Aam = 10.9) and 136.0 ± 26.8 (based on Apm = 5.0). Lastly, as gleaned from this study, based on SC25's N(R<20) = 43 months, one expects SC25's RM = 130.9 ± 39.7. Together, the mean of the five predictions is 148.5 ± 21.1. Assuming SC25 will have RM = 148.5, the 2-cma for SC24 will be 140.4, some 32 units of sunspot number below SC23's value, strongly suggesting that SC24, indeed, marks the beginning of yet another extended interval of low sunspot number minimums and maximums that should persist, at least, through SC26 (and possibly longer; cf. Rigozo, Souza Echer, Evangelista et. al. 2011; Bisoi, Janardhan and Ananthakrishnan 2020).

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