SIMPLE METHODS FOR PREDICTING THE SIZE AND TIMING OF SUNSPOT CYCLE 25

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ABSTRACT

This paper describes several simple methods for estimating the main features of an ongoing solar cycle (SC); in particular, its maximum amplitude (RM), ascent duration (ASC), and period (PER). The current ongoing SC25 had its minimum amplitude (Rm) in December 2019, measuring 1.8. At elapsed time t = 24 months (December 2021), smoothed monthly mean sunspot number R measured 55.7. Comparisons of R with the mean values of Fast-rising-Slowrising and Short-PER-Long-PER cycles strongly suggest that SC25 is best described as being a Slow-rising-Long-PER SC (i.e., Slow-Long), inferring an ASC ≥49 months (i.e., RM occurrence on or after January 2024) and PER ≥131 months (i.e., Rm occurrence for SC26 on or after November 2030). Furthermore, RM >116.4 is expected for SC25 based on the inferred Even-Odd preferential relationship (i.e., during the modern era of sunspot observations SC12-SC24, the Odd-following SC in consecutive Even-Odd cycle pairs has usually had the larger RM, true for 5 of 6 Even–Odd cycle pairs). Slow–Long cycles (i.e., SCs 12, 14, 20, 23, and 24) have an average RM = 132.0 ± 30.6 , ASC = 57 ± 7 months and PER = 138 ± 6 months. Based on the overlap of the geomagnetic minimums (i.e., Aam and Apm) in the vicinity of Rm, one estimates $RM = 145.7 \pm 17.1$ for SC25, a value in stark contrast to the panel prediction of RM =115 peaking in July 2025 ± 8 months. The greatest rate of growth (GRG) in R observed thus far for SC25 is only 5.6 occurring at t = 22 months (October 2021); which, if it holds up, would be the smallest GRG during the modern era (SC12–SC24) and would suggest an RM = 107.6 ± 33.4 and a mean rate of growth (MRG) = 1.7 ± 0.9 for SC25. For comparison, the smallest GRG during SC12–SC24 is 6.5 occurring at t = 39 months associated with SC14, a Slow–Long SC, having RM = 107.1, ASC = 49 months, MRG = 2.09, and PER = 138 months; SC24 had RM = 116.4, ASC = 64 months, GRG = 8.2 at t = 28 months, MRG = 1.78, and PER = 132 months.

INTRODUCTION

Predicting the size and timing of a solar cycle (SC) is crucially important, especially as it relates to the scheduling of space missions. For example, the United States originally intended to return to the Moon in 2024 and ultimately to Mars in the 2030s (Dunbar 2021). However, the moon landing mission has now been delayed to Spring of 2025 at the earliest (Sheetz 2021). Solar activity occurring during these intervals must be closely monitored and forecasted in advance to ascertain possible radiation hazards associated with the occurrences of major flares and coronal mass ejections, especially solar particle events (Hu 2017).

This study describes several simple methods for estimating the main features of a SC: (1) its minimum amplitude occurrence (E(Rm)), (2) its maximum amplitude occurrence (E(RM)), and (3) its minimum and maximum amplitudes (Rm and RM). They are then used to estimate the

size and timing of the current ongoing SC25 using smoothed monthly mean parametric values (i.e., 12–month moving averages), inferred preferential groupings of SCs (e.g., Even–leading and Odd–following; Fast–rising and Slow–rising; and Short–period and Long–period cycle groupings), and inferred statistically important linear regressions (e.g., those against the minimum smoothed monthly mean values of the Aa and Ap geomagnetic indices in the vicinity of E(Rm)).

METHODS AND MATERIALS

Smoothed monthly mean sunspot numbers are taken from http://sidc.oma.be/silso/datafiles and are used to establish E(Rm), E(RM), Rm and RM for SC12–SC25 (i.e., the modern era SCs). From these data, the ascent (ASC; i.e., the elapsed time in months from E(Rm) to E(RM) for each SC) and the period (PER; i.e., the elapsed time in months from E(Rm) for SC *n* to E(Rm) for SC n + 1) for each SC are determined (cf. Wilson 2015, 2019a). Smoothed monthly mean values of the Aa and Ap geomagnetic indices are determined from their monthly mean values taken from https://geomag.bgs.ac.uk/data_service/data/home.html, which are then used to establish the minimum values (Aam and Apm) and occurrences (E(Aam) and E(Apm)) of these geomagnetic parameters in the vicinity of E(Rm) for SC12–SC25.

RESULTS AND DISCUSSION

Table 1 displays selected solar and geomagnetic parameters for SC12–SC25. During the modern era, Rm for an SC has spanned 1.8 (SC25) to 17.8 (SC21) and RM has spanned 107.1 (SC14) to 285.0 (SC19). The mean value of Rm for modern era SCs is 8.1, having a standard deviation (*sd*) equal to 5.2, and the mean value of RM is 175.8, having a *sd* equal to 52.5. Hence, the ± 1 *sd* prediction interval about the mean for Rm and RM is, respectively, 8.1 \pm 5.2 and 175.8 \pm 52.5. For the modern era SCs, Rm values for SC15, SC24, and SC25 fall outside low and Rm values for SC20, SC21, and SC22 fall outside high; while RM values for SC12, SC14 and SC24 fall outside low and RM values for SC19 and SC21 fall outside high. Also given in Table 1 are means and *sds* for various groupings of SCs (i.e., Fast– and Slow–rising; Even– and Odd– numbered; and Long– and Short–PER SCs). The geomagnetic parameters will be discussed later in the text.

Similarly, for the modern era, the ASC of an SC has spanned 38 months (SC22) to 64 months (SC24) in length. For the modern era, the mean value of ASC is 49.8 months, having sd = 8.5 months, thereby yielding the ± 1 sd prediction interval about the mean to be ASC = 49.8 ± 8.5 months. For the modern era SCs, ASC for SC12, SC23 and SC24 have slower ASC (i.e., ASC >58.3 months), while ASC for SC18 and SC22 have faster ASC (i.e., ASC <41.3 months).

Regarding the period PER for a modern era SC, it has spanned 119 months (SC22) to 148 (SC23) months in length. For modern era SCs, the mean PER is found to be 130.2 months, having sd = 9.2 months, thereby yielding the $\pm 1 sd$ prediction interval about the mean to be PER = 130.2 ± 9.2 months. For the modern era SCs, only SC22 has PER outside short (i.e., PER <121 months); SC15 and SC16 have PER = 121 months, which is the lower interval limit for the $\pm 1 sd$ prediction interval about the mean; and SC13 and SC23 have PER outside long (i.e., PER >139.4 months).

SC	Rm	RM	ASC	PER	Aam	Apm	E(Rm)	E(RM)	E(Aam)	E(Apm)	D(Aam)	D(Apm)
12	3.7	124.4	60	135	6.8	-	1878-12	1883-12	1879–01	-	1	-
13	8.3	146.5	46	142	10.7	_	1890-03	1894-01	1890-07	_	4	_
14	4.5	107.1	49	138	6.1	_	1902-01	1906-02	1901–08	_	-5	_
15	2.5	175.7	49	121	8.3	_	1913-07	1917-08	1913-09	_	2	_
16	9.4	130.2	56	121	10.3	_	1923–08	1928-04	1923–11	_	3	_
17	5.8	198.6	43	125	14.1	6.7	1933-09	1937–04	1934–01	1934–06	4	9
18	12.9	218.7	39	122	16.5	10.2	1944-02	1947–05	1945–04	1944–12	14	10
19	5.1	285.0	47	126	17.1	10.8	1954-04	1958-03	1954–10	1954–10	6	6
20	14.3	156.6	49	137	13.9	7.7	1964–10	1968-11	1965–05	1965–05	7	7
21	17.8	232.9	45	126	19.7	10.7	1976-03	1979–12	1976-12	1976–12	9	9
22	13.5	212.5	38	119	17.7	10.0	1986-09	1989–11	1986-12	1986–12	3	3
23	11.2	180.3	63	148	15.9	8.2	1996-08	2001-11	1997–08	1997–08	12	12
24	2.2	116.4	64	132	8.5	3.8	2008-12	2014-04	2009–09	2009–09	9	9
25	1.8	_	_	_	10.9	5.0	2019-12	_	2020-05	2020-05	5	5
mean	81	175.8	49.8	130.2	12.6	8.1						
sd	5.2	52.5	8.5	9.2	4.4	2.6						
Fast#		215 7	42.0	1267	16.0	0.7						
mean	-	45.2	45.0	120.7	10.0	9.7						
sa	-	43.2	5.7	8.0	3.2	1.7						
Slow#												
mean	-	141.5	55.7	133.1	10.0	6.6						
sd	-	29.3	6.8	9.7	3.7	2.4						
Even												
mean	8.6	152.3	50.7	129.1	11.4	7.9						
sd	5.1	45.9	10.0	8.2	4.7	3.0						
Odd												
mean	7.5	203.2	48.8	131.3	13.8	8.3						
sd	5.6	49.2	7.2	10.9	4.1	2.5						
L ong#												
mean	_	138.6	55.2	138.7	10.3	6.6						
sd	_	27.6	8.0	5.6	3.9	2.4						
Short#												
mean	_	207.7	45.3	122.9	14.8	9.7						
sd	_	48.1	6.2	2.8	4.2	9.7						

Table 1. Solar and geomagnetic parameters for SC12–SC25.

Notes:

means SC25 values excluded

RM is maximum smoothed monthly mean sunspot number

ASC is the ascent duration in months, E(RM) - E(Rm)

PER is the period, the elapsed time in months between $E(Rm) \mbox{of } SC(n)$ and $E(Rm) \mbox{ of } SC(n+1)$

Aam is the minimum smoothed monthly mean aa value

Apm is the minimum smoothed monthly mean Ap value

E(Rm) is the epoch of minimum smoothed monthly mean sunspot number E(RM) is the epoch of maximum smoothed monthly mean sunspot number

E(Aam) is the epoch of minimum smoothed monthly mean aa value

E(Apm) is the epoch of minimum smoothed monthly mean Ap value

D(Aam) is the elapsed time in months between E(Aam) and E(Rm), where positivevalues mean E(Aam) follows E(Rm)

D(Apm) is the elapsed time in months between E(Apm) and E(Rm), where positivevalues mean E(Apm) follows E(Rm)

Comparisons of RM against ASC and PER are shown in Figure 1. Regarding the scatterplot of RM versus ASC (Figure 1a), notice that the upper-left and lower-right quadrants are the most populated quadrants, accounting for 10 of 13 SCs. In the figure, the numbers represent the SC numbers; filled circles represent Short-period SCs; and unfilled circles represent Long-period SCs, where a Long-period SC is one having PER >131 months. The vertical and horizontal lines represent the median values of ASC (49 months) and RM (175.7), respectively. The probability of obtaining the observed result ($P_o = 0.0020$), or one more suggestive of a departure from independence (chance), is computed to be P = 0.0061 (i.e., 0.61%) for RM versus ASC, based on the Fisher's exact test for 2×2 contingency tables (Langley 1970). Hence, one infers there exists a rather strong statistical relationship between RM and ASC (i.e., the Waldmeier effect; Waldmeier 1935; Kiepenheuer 1953, Hathaway 2015, Wilson 2015, 2019a), associating large RM with Fast-rising ASC and small RM with Slow-rising ASC. The exceptions are SC13, SC15, and SC23. Furthermore, one infers that SCs of large RM and Fast-rising ASC also tend to be SCs of Short-PER, while SCs of small RM and Slow-rising ASC tend to be SCs of Long-PER. Therefore, if an ongoing SC is believed to be a Fast-rising SC, one infers that it likely will have a large RM and be Short-PER, while if it is believed to be a Slow-rising SC, one would infer that it likely will have a small RM and be Long-PER. From Table 1, Fast-rising ASC SCs have ± 1 sd prediction intervals for RM, ASC and PER equal to 215.7 ± 45.2 , 43.0 ± 3.7 months, and 126.7 ± 8.0 months, while Slow-rising ASC SCs have ± 1 sd prediction intervals of $141.5 \pm$ 29.3, 55.7 ± 6.8 months, and 133.1 ± 9.7 months, respectively.



Figure 1. (a) RM versus ASC for SC12–SC24; (b) RM versus PER for SC12–SC24.

Regarding the scatterplot of RM versus PER (Figure 1b), as in Figure 1a the upper–left and lower–right quadrants are the most populated, accounting for 11 of 13 SCs. The filled circles represent Fast–rising SCs and the unfilled circles represent Slow–rising SCs. Notice the gap in PER occurring between 127 and 131 months, which encompasses the mean PER (130.2 months; it should be noted that the gap is seen in all SC1–SC24; cf. Wilson 2015). Based on the Fisher exact test for 2×2 contingency tables, P = 0.0047 (i.e., 0.47%), suggesting a rather strong statistical relationship to exist between RM and PER. Hence, one infers that Short–PER SCs tend to have large RM and be Fast–rising, while Long–PER SCs tend to have small RM and be Slow–rising. The exceptions are SC16 and SC23. From Table 1, Short–PER SCs have ± 1 *sd* prediction intervals for RM, ASC and PER equal to 207.7 \pm 48.1, 45.3 \pm 6.2 months, and 122.9 \pm 2.8 months, while Long–PER SCs have ± 1 *sd* prediction intervals of 138.6 \pm 27.6, 55.2 \pm 8.0 months, and 138.7 \pm 5.6 months, respectively.

Another aspect of modern era SCs is the apparent Even–leading–Odd–following RM association (i.e., for consecutive Even–Odd cycle pairs, the Even–leading SC generally has smaller RM in comparison to the Odd–following SC, true for 5 of the 6 Even–Odd modern era cycle pairs). Only the most recent pairing SC22/23 did not adhere to this behavior. Presuming that the Even–Odd association remains valid for SC24/25, because RM(SC24) = 116.4, one infers that RM(SC25) > 116.4. (The Even–Odd association was true only for 3 of the 6 cycle pairs for the earlier SC0–SC11.)

Figure 2 shows the scatterplot of mean RMs of Odd–following SCs versus mean RMs of Even–leading SCs for the 6 modern era cycle pairs. From Table 1, one finds that Even–leading SCs have a mean RM and *sd* of 152.3 and 45.9, respectively, while Odd–following SCs have a mean RM and *sd* of 203.2 and 49.2. Excluding SC22/23, which does not adhere to the Even–Odd association, the mean RM and *sd* for Even and Odd SCs are 147.4 and 43.6 and 207.7 and 53.5, respectively. The mean difference in RM and *sd* for the 5 SC pairs that adhere to the Even–Odd association are, respectively, 60.3 and 21.7. Hence, presuming SC24/25 adheres to the Even–Odd association, RM(SC25) = $116.4 + 60.3 \pm 21.7$, or about 176.7 ± 21.7 .



Figure 2. Mean RM(Odd-following) versus mean RM(Even-leading) for SC12/13 to SC22/23.

Also shown in Figure 2 is the inferred linear regression y between RM Odd–following versus RM Even–leading SCs for the 5 cycle pairs that adhere to the Even–Odd SC paradigm (i.e., SC22/23 is excluded). The inferred regression equation is y = 41.38 + 1.1286x, where y is the inferred RM Odd–following value and x is the RM observed Even–leading value. The inferred regression has a coefficient of correlation r = 0.92, a coefficient of determination $r^2 = 0.85$ (meaning that 85% of the variance can be explained by the inferred regression, a standard error of estimate se = 24.25, a t statistic equal to 4.09 (inferring that the regression is statistically important at the 5% level of statistical significance based on the sample size) and a confidence level cl >95%. Presuming SC pair 24/25 adheres to the Even–Odd association, one infers RM(SC25) = 172.9 ± 24.3 (i.e., the ± 1 se prediction interval).

Figure 3 compares the growth in smoothed monthly mean sunspot number R of SC25 with the mean growth in R for Odd–numbered SCs (Figure 3a) and the relative month–to–month rate of growth (RG) of SC25 with the mean RG for Odd–numbered SCs (Figure 3b). Clearly, SC25's observed behavior is well below that expected for Odd–numbered SCs. For December 2021 (t =24 months), R(SC25) = 55.7, a value –1.47 *sd* below the mean for Odd–numbered SCs (R(Odd) = 104.5 at t = 24 months. Likewise, the maximum RG for SC25 observed thus far is 5.6 (at t = 22months), a value below the mean RG for Odd–numbered SCs. The RM values and Greatest Rate of Growth (GRG) values are shown to the right in Figures 3a and b, and the epoch of those values (E(RM) and E(GRG)) are shown near the top. Clearly, both R(SC25) and RG(SC25) are well below the lowest observed values for Odd–numbered SCs (i.e., 146.5 and 7.6 for SC13, respectively). (Table 2 provides the observed values used to generate Figures 3a and 3b.)



Figure 3. Comparison of (a) R for Odd–numbered SCs and SC25; (b) RG for Odd–numbered SCs and SC25.

t	Odd	SC25	D(Odd)	RG(Odd)	RG(SC25)
0	8.5(5.5)	1.8	-1.22	0.4	0.4
1	8.9(5.6)	2.2	-1.20	0.7	0.5
2	9.6(5.4)	2.7	-1.28	1.1	0.3
3	10.7(4.8)	3.0	-1.60	1.1	0.6
4	11.8(4.5) 12.0(4.7)	3.6	-1.82	1.2	2.0
5	13.0(4.7) 13.8(4.6)	5.0 7.9	-1.37 -1.28	0.8	2.5
7	14.9(4.5)	9.0	-1.31	2.2	0.5
8	17.1(5.1)	9.5	-1.49	2.6	1.0
9	19.7(6.4)	10.5	-1.44	3.2	1.4
10	22.9(7.9)	11.9	-1.39	3.1	1.7
11	26.0(8.8)	13.6	-1.41	3.4	1.7
12	29.4(9.6)	15.3	-1.47	4.4	2.0
13	33.8(11.0)	17.3	-1.50	4.5	1.7
14	38.3(12.8)	19.0	-1.59	4.3	2.7
15	42.6(13.6)	21.7	-1.54	5.2	3.1
10	47.8(13.1) 54.8(17.9)	24.8	-1.52 -1.62	7.0	1.0
18	62.6(19.7)	25.8	-1.02 -1.78	7.0 8.2 <i>&</i>	3.7
19	70.8(21.1)	31.3	-1.87	7.5	4.0
20	78.3(22.2)	35.3	-1.94	6.5	4.7
21	84.8(23.6)	40.0	-1.90	6.5	5.0
22	91.3(26.3)	45.0	-1.76	6.6	5.6
23	97.9(30.1)	50.6	-1.57	6.6	5.1
24	104.5(33.2)	55.7	-1.47	6.3	
25	110.8(36.0)			6.1	
26	116.9(39.7)			5.9	
27	122.8(43.2)			6.4	
28	129.2(43.4)			4.7	
29	133.9(43.1) 127.7(44.8)			3.8	
30	137.7(44.8) 130.8(47.1)			2.1	
32	139.8(47.1) 142.7(49.5)			5.1	
33	142.7(49.3) 147.8(51.8)			4.6	
34	152.4(52.2)			4.9	
35	157.3(52.7)			5.4	
36	162.7(55.2)			4.7	
37	167.4(56.6)			4.3	
38	171.7(57.4)			6.1	
39	177.8(58.4)			3.7	
40	181.5(59.0)			4.1	
41	185.6(58.5)			3.4	
42	189.0(57.6)			4.1	
43	193.1(55.8) 106.5(52.2)			3.4	
44	190.3(33.3) 107.0(51.8)			0.3	
45 46	197.0(31.8)			0.4	
47	197.7(51.4)M			-1.4	
48	196.3(49.5)			-2.6	
49	193.7(47.1)			-2.8	
50	190.9(45.5)			-2.2	
51	188.7(45.7)			-1.7	
52	187.0(46.4)			-3.2	
53	183.8(47.0)			-2.7	
54	181.1(47.5)			-1.0	
55	180.1(46.9)			-1.0	
56	179.1(47.4)			-1.5	
57	177.6(47.0)			-1.5	
58	176.1(47.2)			-1.7	
59	174.4(48.2)			-2.3	
60	170.7(47.9)			-1.4	
01 62	1/0./(4/.1)			-1.2	
62	109.3(43.7)			-1.8	
64	107.7(43.7) 164 8(42 4)			-2.9	
65	162 9(41 1)			-0.4	
66	162.5(40.6)			-2.6	

Table 2. $\langle R \rangle$ (*sd*) values for t = 0-72 months for Odd–numbered SCs and SC25.

t	Odd	SC25	D(Odd)	RG(Odd)	RG(SC25)
67	159.9(41.0)			-4.9	
68	155.0(40.6)			-4.3	
69	150.7(41.0)			-3.9	
70	146.8(40.8)			-4.6	
71	142.2(39.9)			-3.2	
72	139.0(38.6)				

Notes: & means greatest RG <R> means mean sunspot number RD means difference RG means rate of growth M means maximum value of <R>

Figure 4 compares the monthly R and RG for SC25 with the mean R and RG for Fast– and Slow–rising SCs (Figures 4a and b, respectively). Again, observed RM and GRG values for SC12–SC24 appear to the right and their epochs of occurrence are shown near the top. Clearly, SC25 behavior appears more closely matched to that for Slow–rising SCs than for Fast–rising SCs. Presently, its R value of 55.7 (at t = 24 months) and GRG value of 5.6 (at t = 22 months) are below the smallest observed values for modern era SCs. For example, SC12 had RM = 124.4 at t = 60 months and GRG = 6.8 at t = 14 months and SC14 had RM = 107.1 at t = 49 months and GRG = 6.5 at t = 39 months. The most recent SC23 and SC24 had RM = 180.3 at 63 months and 116.4 at t = 64 months, respectively, and GRG = 8.8 at t = 37 months and 8.2 at t = 28 months, respectively. Hence, SC25's behavior suggests that it likely has another, perhaps, 2–3 years of growth before attaining RM (and probably GRG), presuming that it is a Slow–rising SC. (Table 3 provides the observed values used to generate Figures 4a and 4b.)



Figure 4. Comparisons of (a) Mean R for Fast– and Slow–rising SCs and SC25; (b) Mean RG for Fast– and Slow–rising SCs and SC25.

	Fast	Slow	SC24	SC25	D(Fast)	D(Slow)	RG(Fast)	RG(Slow)	RG(SC25)
0	10.6(5.0)	6.8(4.8)	2.2	1.8	-1.76	-1.04	0.6	0.3	0.4
1	11.2(5.1)	7.1(4.9)	2.5	2.2	-1.76	-1.00	1.1	0.6	0.5
2	12.3(4.8)	7.7(5.1)	2.7	2.7	-2.00	-0.98	0.9	1.1	0.3
3	13.2(4.4)	8.8(5.3)	2.9	3.0	-2.32	-1.09	1.2	1.1	0.6
4	14.4(4.3)	9.9(5.3)	3.3	3.6	-2.51	-1.19	1.6	1.0	2.0
5	16.0(4.8)	10.9(5.5)	3.5	5.6	-2.17	-0.96	0.7	1.1	2.3
6	17.3(5.6)	12.0(6.1)	4.1	7.9	-1.67	-0.67	1.2	1.3	1.1
7	18.5(5.9)	13.8(6.5)	5.5	9.0	-1.61	-0.74	2.2	1.8	0.5
8	20.7(6.2)	15.8(6.6)	7.4	9.5	-1.81	-0.95	3.2	2.0	1.0
9	23.9(6.8)	17.7(6.6)	9.5	10.5	-1.97	-1.09	3.7	1.9	1.4
10	27.6(7.8)	19.6(6.7)	10.9	11.9	-2.01	-1.15	3.7	1.9	1.7
11	31.3(8.9)	21.5(7.0)	11.7	13.6	-1.99	-1.13	3.8	1.9	1.7
12	35.1(9.7)	24.1(7.4)	12.7	15.3	-2.04	-1.19	4.2	3.3	2.0
13	39.3(11.1)	27.4(8.1)	14.0	17.3	-1.98	-1.25	4.4	3.3	1.7
14	43.7(13.0)	30.7(8.6)	16.1	19.0	-1.90	-1.36	4.8	3.9	2.7
15	48.5(14.2)	34.6(9.4)	18.5	21.7	-1.89	-1.37	5.6	4.1	3.1
16	54.1(15.7)	38.7(10.4)	20.8	24.8	-1.87	-1.34	6.5	4.4	1.0
17	60.6(18.3)	43.1(11.2)	23.1	25.8	-1.90	-1.54	7.8	4.3	1.8
18	68.4(20.3)	47.4(12.5)	24.6	27.6	-2.01	-1.58	8.4	3.5	3.7
19	76.8(21.6)^	50.9(14.3)	25.2	31.3	-2.11	-1.37	8.1	5.5	4.0
20	84.9(22.7)	56.4(16.1)	26.4	35.3	-2.19	-1.31	8.2	4.0	4.7
21	93.1(24.6)	60.4(16.9)	29.5	40.0	-2.16	-1.21	8.3	5.6&	5.0
22	101.4(28.1)	66.0(17.6)	34.5	45.0	-2.01	-1.19	8.9	5.5	5.6
23	110.3(31.8)	71.5(18.5)	39.1	50.6	-1.81	-1.13	9.0&	4.3	5.1
24	119.3(33.9)	75.8(19.3)	42.5	55.7	-1.88	-1.04	8.0	3.9	
25	127.3(34.9)	79.7(19.3)	45.7				7.6	3.3	
26	134.9(37.4)	83.0(19.4)^	48.8				8.1	2.9	
27	143.0(40.7)%	85.9(19.1)	53.8				7.4	4.3	
28	150.(41.6)	90.2(19.8)	61.1				5.6	4.3	
29	156.0(41.5)	94.5(20.4)	69.3				5.5	3.1	
30	161.5(42.7)	97.6(19.9)	77.2				4.7	2.3	
31	165.8(43.3)	99.9(18.5)	83.6				4.8	2.3	
32	170.6(44.4)	102.2(19.0)	86.3				6.9	2.1	
33	177.5(46.6)	104.3(21.2)	86.6				4.6	1.6	
34	182.1(43.7)	105.9(21.8)	87.4				4.9	1.6	
35	187.0(42.1)	107.5(21.2)	89.4				4.8	2.2	
36	191.8(43.5)	109.7(21.2)	92.5				4.4	2.5	
37	196.2(45.1)	112.2(23.1)	95.5				4.4	2.8	
38	200.6(45.2)	115.0(26.2)	98.1				3.1	1.8	
39	203.7(45.0)	117.8(27.2)	98.3				1.9	1.0	
40	205.6(44.4)	118.8(26.6)	95.1				2.6	1.8	
41	208.2(44.5)	120.6(27.5)	90.9				0.9	1.7	
42	209.1(44.8)	122.3(29.7)	86.6				-0.3	1.6	
43	208.8(46.0)	123.9(32.0)	84.5				0.5	1.5	
44	209.3(46.1)	125.4(33.6)	85.1				-0.7	1.5	
45	208.6(45.9)	124.9(33.5)	85.3				-0.7	0.8	
46	207.9(46.7	125.7(34.6	85.8				-1.1	1.4	
47	206.8(47.6)	127.1(36.7)	87.7				-2.3	1.6	

Table 3. <R>(sd) values for t = 0-72 months for Fast- and Slow-rising SCs and SC25.

	Fast	Slow	SC24	SC25	D(Fast)	D(Slow)	RG(Fast)	RG(Slow)	RG(SC25)
48	204.5(46.6)	128.7(37.7)	88.1				-2.3	1.6	
49	202.0(44.2)	129.2(36.9)	86.8				-2.4	-0.7	
50	199.6(42.4)	128.5(35.5)	86.1				-1.8	-1.6	
51	197.8(42.8)	126.9(35.6)	84.4				-0.6	-1.2	
52	197.2(43.7)	125.7(36.0)	84.3				-0.8	-1.2	
53	196.4(43.7)	124.7(33.4)	87.0				0.0	0.2	
54	196.4(43.4)	124.9(29.7)	90.9				0.9	2.4	
55	197.3(43.3)	127.3(26.7)	94.6				-0.7	2.1	
56	196.6(43.8)	129.4(24.2)	99.0				-2.7	1.3	
57	193.9(43.6)	130.7(23.1)	104.6				-3.1	1.2	
58	190.8(44.7)	131.9(23.7)	107.0				-2.1	-0.5	
59	188.7(45.5)	131.4(24.7)	106.9				-2.5	-1.7	
60	186.2(44.2)	129.7(26.4)	107.6				-2.3	-1.3	
61	183.9(43.0)	128.4(27.6)	109.3				-2.2	-0.1	
62	181.7(41.7)	128.3(27.3)	110.5				-2.9	1.3	
63	178.8(39.4)	129.6(27.0)	114.3				-4.0	0.1	
64	174.8(38.7)	129.7(26.1)	116.4				-4.6	-1.0	
65	170.2(38.9)	128.7(26.5)	115.0				-3.4	-1.5	
66	166.8(38.4)	127.2(28.7)	114.1				-3.8	-3.8	
67	163.0(38.2)	123.4(30.7)	112.6				-5.0	-4.2	
68	158.0(37.7)	119.2(31.9)	108.3				-4.3	-2.8	
69	153.7(38.4)	116.4(31.6)	101.9				-4.2	-2.3	
70	149.5(38.6)	114.1(30.2)	97.3				-5.1	-2.2	
71	144.4(38.3)	111.9(28.0)	94.7				-3.8	-1.9	
72	140.6(37.8)	110.0(25.5)	92.2						

Notes:

^ means first occurrence of R value = 100 or higher; for Fast it is SC19 and forSlow it is SC20
% means first occurrence of R value = 200 or higher; for fast it is SC19; no SlowSC ever achieved 200 or higher
& means greatest RG observed thus far
<R> means mean sunspot number RD means difference

RG means rate of growth

Figure 5 displays the R values for the northern and southern hemispheres for SC23 (Figure 5a), SC24 (Figure 5b) and SC25 (Figure 5c). For t = 24 months, SC23 had R = 93.5, R(NH) = 45.7 and R(SH) = 47.8; SC24 had R = 42.5, R(NH) = 28.3 and R(SH) = 14.2; and SC25 had R = 55.7, R(NH) = 25.9 and R(SH) = 29.8. For SC23, its R(SH) clearly is double peaked, peaking at t = 67 months. For SC24, R plainly is double peaked, with R(NH) peaking early at t = 32 months and R(SH) peaking at t = 64 months when R peaks. Presently, for SC25 one cannot differentiate whether or not R, R(NH) and R(SH) will be double-peaked.



Figure 5. R values for northern and southern hemispheres for (a) SC23, (b), SC24, and (c) SC25.

Table 4 gives the mean rate of growth (MRG = (RM – Rm)/ASC)), GRG and *t* values for SC12–SC24, where *t* is the elapsed time in months from E(Rm) to E(GRG) and *t*' is the elapsed time in months from E(GRG) to E(RM). Also given is the ASC–PER class for each SC (i.e., Slow–Long, Fast–Long, Slow–Short, Fast–Short). Overall, MRG has a mean and *sd* of 3.53 and 1.45, respectively; GRG has a mean and *sd* of 10.3 and 2.9, respectively; *t* has a mean and *sd* of 27 and 9 months, respectively; and *t*' has a mean and *sd* of 23 and 12 months, respectively. Notice however, that SCs tend to be divided into 2 major groupings: Fast–Short and Slow–Long, accounting for 10 of the 13 modern era SCs. SC13, SC15 and SC16 are statistical outliers with respect to this inferred preferential grouping of Fast–Short and Slow–Long. Fast–Short cycles have means/*sds* of 5.15/0.56, 12.4/2.2, 26/7 and 16/8 for MRG, GRG, *t* and *t*', while Slow–Long cycles have means/*sds* of 2.29/0.48, 7.9/1.2, 28/10 and 29/13. Hence, if SC25 is indeed a Slow–Long SC, then one expects 1.81 ≤ MRG ≤ 2.77, 6.7 ≤ GRG ≤ 9.1, 18 ≤ *t* ≤ 38, and 16 ≤ *t*' ≤ 42, inferring E(GRG) any time between *t* = 18 months (June 2021) and 38 months (February 2023) and E(RM) any time after December 2023 and before May 2025.

SC	Rm	RM	ASC	MRG	GRG	t	ť	Comment
12	3.7	124.4	60	2.01	6.8	14	46	Slow-Long
13	8.3	146.5	46	3.00	7.6	16	30	Fast-Long
14	4.5	107.1	49	2.09	6.5	39	10	Slow-Long
15	2.5	175.7	49	3.53	14.2	43	6	Slow-Short
16	9.4	130.2	56	2.16	10.3	22	34	Slow-Short
17	5.8	198.6	43	4.48	10.6	39	4	Fast-Short
18	12.9	218.7	39	5.28	10.7	23	16	Fast-Short
19	5.1	285.0	47	5.96	15.3	22	25	Fast-Short
20	14.3	156.6	49	2.90	9.2	22	27	Slow-Long
21	17.8	232.9	45	4.78	11.4	27	18	Fast-Short
22	13.5	212.5	38	5.24	14.2	21	17	Fast-Short
23	11.2	180.3	63	2.68	8.8	37	26	Slow-Long
24	2.2	116.4	64	1.78	8.2	28	36	Slow-Long
			mean	3.53	10.3	27	23	
			sd	1.45	2.9	9	12	
			Fast					
			mean	4.79	11.6	25	18	
			sd	1.01	2.8	8	9	
			Slow					
			mean	2.45	9.1	29	26	
			sd	0.62	2.6	11	14	
			Even					
			mean	3.07	9.4	24	27	
			sd	1.54	2.7	8	13	
			Odd					
			mean	4.07	11.3	31	18	
			sd	1.23	3.0	11	11	

Table 4. Rm, RM, ASC, MRG, GRG, t and t' values for SC12–SC24.

Long				
mean	2.41	7.9	26	29
sd	0.51	1.1	11	12
Short				
mean	4.49	12.4	28	17
sd	1.28	2.1	9	10

Notes:

Rm is the minimum smoothed monthly mean sunspot number RM is the maximum smoothed monthly mean sunspot number

ASC is the ascent duration in months from E(Rm) to E(RM)MRG means mean rate of growth where MRG = (RM-Rm)/ASC

GRG means greatest rate of growth w

t means the elapsed time in months from E(Rm) to E(GRG)

t' means the elapsed time in months from E(GRG) to E(RM)

Figure 6 displays scatterplots of (a) MRG versus GRG; (b) RM versus GRG; and (c) RM versus MRG. Each scatterplot is found to be highly statistically significant, based on both the Fisher's exact test and linear regression analysis. From Figure 6a, one finds that all SCs having GRG <10.3 are Long–PER SCs and have MRG \leq 3.00, while nearly all SCs having GRG \geq 10.3 are Short–PER SCs and have MRG >3.00 (with one exception, SC16). From Figure 6b, one finds that all SCs having GRG \leq 10.3 are Long–PER SCs and have RM \leq 180.3, while nearly all SCs having GRG \geq 10.3 are Short–PER SCs and have RM \geq 175.7 (with one exception, SC16). From Figure 6c, one finds that nearly all SCs having MRG \leq 3.00 are Long–PER SCs (with one exception, SC16), while all SCs having MRG \geq 3.00 are Short–PER SCs. Hence, by monitoring the RG and GRG for an ongoing SC, one likely can determine its ASC–PER class.



Figure 6. (a) MRG vs. GRG; (b) RM vs. GRG; and (c) RM vs. MRG for SC12–SC24. An S after the SC number indicates a Short–PER SC and an L indicates a Long–PER SC.

Figure 7 displays the running (a) RG and (b) MRG for SC25, the current ongoing SC. Thus far, SC25 has a potential GRG = 5.6 occurring at t = 22 months, a value well within the timeframe one expects to see a cycle's GRG. If indeed GRG = 5.6 for SC25, then one predicts that SC25 will be a Slow–Long SC, having MRG <3.00 and RM ≤180.3. Using the inferred linear regressions from Figures 6a and 6b, one infers MRG = 1.70 ± 0.94 (or MRG <2.64, with confidence level cl = 84.26%) and RM = 107.6 ± 33.4 (or RM <141). However, if GRG >5.6 for SC25, occurring later, then a higher MRG and RM would be expected. The running MRG continues to increase in value, being 2.25 at t = 24 months. (Recall from Table 4 that GRG and for SC23 and SC24 measured 8.8 and 8.2; E(GRG) occurred at t = 37 months and t = 28 months; and MRG measured 2.68 and 1.78, respectively. The largest running MRG prior to E(RM) for SC23 and SC24 measured 3.92 and 2.70 and occurred at t = 39 months and t = 32 months, respectively.)



Figure 7. (a) Running RG and (b) Running MRG for SC25 for t = 0-24 months.

Now it has long been known that the minimum value of the Aa and Ap geomagnetic indices in the vicinity of E(Rm) can be used to infer RM for the ongoing SC (e.g., Ohl 1966; Wilson 1990, 2019b; Wilson and Hathaway 2006). Table 1 provides the Aam and Apm values and E(Aam) and E(Apm) dates for SC12–SC25 and the delays (D(Aam) and D(Apm), in months) of their occurrences relative to E(Rm). (Only SC14 had an E(Aam) prior to E(Rm). E(Aam) and E(Apm) have always occurred concurrently, except for SC17 and SC18.)

Figure 8 displays the scatter plots of RM versus Aam (Figure 8a) and RM versus Apm (Figure 8b). In the plot, Fast-ASC cycles are denoted as filled circles and Slow-ASC cycles are denoted as unfilled circles. SC25 values are denoted by the upside-down unfilled triangle along the x-axis and S and L denotes cycles of Short- and Long-PER, respectively. The numbers denote the SC number. Shown in the figures are the inferred linear regression y, the inferred linear regression equation, the coefficient of linear regression r, the coefficient of determination r^2 , the standard error of estimate se, and the t value and confidence level cl, as well as the result of the Fisher's exact test for the observed 2×2 contingency tables (given as P_o and P). Notice from Figure 8a that the Aam for SC25 is larger than the Aam values for SC12–SC16 and SC24. Similarly, notice from Figure 8b that the Apm value for SC25 is larger than the Apm value for SC24. From Figure 8a the Aam = 10.9 suggests that RM for SC25 will be in the lower-left quadrant (<175.7). Based on the inferred linear regression, one estimates that $RM = 157.6 \pm$ 29.0, or $128.6 \le \text{RM} \le 186.6$ (*cl* = 84.26%) for SC25. Similarly, from Figure 8b the Apm = 5.0 suggest that RM for SC25 will be in the lower-left quadrant (<205.6). Based on the inferred linear regression, one estimates that $RM = 136.0 \pm 26.8$, or $109.2 \le RM \le 162.8$ (*cl* = 84.26%). The overlap in the two geomagnetic predictions of RM for SC25 is $128.6 \le RM \le 162.8$, or RM $= 145.7 \pm 17.1.$



Figure 8. (a) RM versus Aam for SC12–SC24; (b) RM versus Apm for SC17–SC24.

Predicting solar activity has been a subject of investigation since sunspots were first observed (e.g., Pesnell 2012; Hathaway 2015; Usoskin 2017; and Petrovay 2020). Many techniques have been developed through the years to accomplish the aim of predicting the various features of an SC, some more successful than others. Regarding predictions for the current ongoing SC25:

(1) Rigozo, Souza Echer at al. (2011) have predicted that SC25 will be weak with a PER of 118 months and RM = 132.1 in April 2023.

(2) Helal and Galal (2013) have predicted RM = 118.2 and ASC = 4.0 yrs.

(3) Cameron, Jiang and Schüssler (2016) have predicted SC25 to be of moderate amplitude not much higher than what was seen in SC24.

(4) Kakad, Kakad and Ramesh (2017) have predicted SC25 will be a significantly weaker SC, comparable to solar activity observed during the Dalton minimum ($RM = 63 \pm 11.3$).

(5) Bhowmik and Nandy (2018) have predicted RM(SC25) = 118 with a range of 109–139 peaking in 2024 with a range of 2023–2025.

(6) Upton and Hathaway (2018) have predicted that SC25 will be weak in strength preceded by a long extended minimum.

(7) Pesnell and Schatten (2018) have predicted SC25 to have RM = 135 ± 25 peaking about 2015.2 \pm 1.5 yrs.

(8) Labonville, Charbonneau and Lemerle (2019) have predicted RM = 89 + 29/-14 for SC25, peaking in 2025.3 +0.89/-1.05 with a 6-month onset delay in R(NH) but having a peak R(NH) 20% higher than R(SH).

(9) Bisoi, Janardhan and Ananthakrishnan (2019) have predicted SC25 to have $RM = 134 \pm 11$ or 131 ± 11 , assuming Rm occurrence in 2020 or 2021, respectively, and suggest that SC25 will be stronger than SC24 and a little weaker than SC23, arguing that SC25 will be another "mini solar maximum."

(10) Pesnell (2020) examined lessons learned from predictions of SC24, applying them to predictions for SC25. In particular, he notes that, so far, 16 climatological or statistical forecasts of SC25 have been published and they span the range from low to high RM, although the predictions for SC25 have a smaller spread than was seen for SC24 but are grouped around the same amplitude that was seen in SC24. Furthermore, he added that the Sun's magnetic field suggests that SC25 will have an RM slightly larger than was seen in SC24.

(11) McIntosh, Chapman et al., (2020) suggest that SC25 could have an RM that rivals the top few since the sunspot record began, perhaps being the strongest SC ever observed.

(12) Nandy (2021) has noted that physics-based predictions for SC25 have converged and indicates that it will be a weak- to moderate-sized SC.

(13) Guo, Jiang and Wang (2021) have predicted SC25 to be about 10% stronger than SC24, having an RM = 126, and suggest that SC25 will not enter a Maunder–like grand solar minimum as so many researchers have predicted.

(14) Janssens (2021) has predicted SC25 to have an RM = 118 ± 29 using polar faculae observations.

(15) Chowdhury, Jain et al. (2021) have predicted SC25 to have $RM = 100.21 \pm 15.06$, perhaps peaking in April 2025 ± 6.5 months, stating that SC25 will be weaker than or comparable to SC24 and suggesting that the Sun is approaching a global minimum.

(16) Rahmanifard, Jordan et al. (2021) have predicted SC25 to be as weak or weaker than SC24 and that the Sun is entering a secular minimum that will last 2 cycles (SC25–SC26).

(17) Carrasco and Vaquero (2021) have suggested a maximum for SC25 that will be small-moderate and similar to those observed in SC7 and SC24.

(18) Lu, Xiong et al. (2022) have predicted that SC25 will have a single peak with RM = 145.3 occurring in October 2024.

(19) Ivanov (2022) has predicted that SC25 will have $RM = 181 \pm 46$ peaking at 2024.2 ± 1.0 with a probability of 92%; and Arregui (2022) has predicted SC25 to have RM = 184 + 25/-22. (A running summary of SC25 observations/predictions can be found at https://en.wikipedia.org/wiki/Solar_cycle_25#:~:text=Cycle 25 Predictions.)

This study suggests that SC25 is a Slow–rising cycle that likely will also be one of Long PER, inferring RM >116.4, ASC ≥49 months and PER ≥131 months. Hence, E(RM) for SC25 is expected on or after January 2024 and onset for SC26 on or after November 2030. Presuming SC25 is indeed a Slow–Long SC, one expects RM = 132.0 ± 30.6, ASC = 57 ± 7 months and PER = 138 ± 6 months. Based on the overlap of the geomagnetic minimums (Aam and Apm) in the vicinity of Rm, one expects RM = 145.7 ± 17.1 . Thus far, the GRG for SC25 measures only 5.6 (at *t* = 22 months; this value is expected to actually be larger and occur later), the smallest such value in the modern era (SC12–24). If that value holds up, then RM = 107.6 ± 33.4 and MRG = 1.7 ± 0.9 . For comparison, Slow–Long SCs have mean GRG, MRG, *t* and *t*' equal to 7.9 ± 1.2 , 2.29 ± 0.48 , 28.0 ± 10.4 months and 29.0 ± 13.3 months, respectively. Presently, R(NH) and R(SH) are continually rising and are of near equal size, with R(SH) being slightly larger.

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